Automation and Sectoral Reallocation

Online Appendix

Dennis Hutschenreiter[1] Tommaso Santini[2]

Eugenia Vella[3]

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[1]Universitat Autònoma de Barcelona and Barcelona GSE. *e-mail:* Dennis.Hutschenreiter@uab.cat

[2]Universitat Autònoma de Barcelona and Barcelona GSE. *e-mail:* tommaso.santini@uab.cat.

[3]Athens University of Economics and Business, Greek and European Economy Observatory at ELIAMEP (Hellenic Foundation for European and Foreign Policy), and Fundació MOVE (Markets, Organizations and Votes in Economics). *e-mail:* evella@aueb.gr

1 The maximization problem of the household

The Lagrangian is given by:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\eta}}{1-\eta} + \Phi \frac{\left[1 - u_t - n_t^M - n_t^S\right]^{1-\varphi}}{1-\varphi} \right.$$
$$- \lambda_{ct} \left[c_t + k_{t+1} - (1-\delta)k_t - r_t k_t - w_t^M n_t^M - w_t^S n_t^S - \bar{b}_t u_t - \Pi_t^M - \Pi_t^S\right]$$
$$- \lambda_{n^M t} \left[n_{t+1}^M - \left(1 - \sigma^M\right) n_t^M - \psi_t^{hM} s_t u_t\right]$$
$$\left. - \lambda_{n^S t} \left[n_{t+1}^S - \left(1 - \sigma^S\right) n_t^S - \psi_t^{hS} (1 - s_t) u_t\right] \right\}.$$

The first-order conditions with respect to c_t, k_{t+1}, n_{t+1}^M, n_{t+1}^S, u_t, and s_t are,
[wrt c_t]

$$c_t^{-\eta} - \lambda_{ct} = 0, \tag{A.1}$$

[wrt k_{t+1}]

$$- \beta^t \lambda_{ct} + \beta^{t+1} E_t \left[\lambda_{ct+1}(1 - \delta + r_{t+1})\right] = 0, \tag{A.2}$$

[wrt n_{t+1}^M]

$$- \beta^t \lambda_{n^M t} + \beta^{t+1} E_t \left[-\Phi l_{t+1}^{-\varphi} + \lambda_{ct+1} w_{t+1}^M + \lambda_{n^M t+1}(1 - \sigma^M)\right] = 0, \tag{A.3}$$

[wrt n_{t+1}^S]

$$- \beta^t \lambda_{n^S t} + \beta^{t+1} E_t \left[-\Phi l_{t+1}^{-\varphi} + \lambda_{ct+1} w_{t+1}^S + \lambda_{n^S t+1}(1 - \sigma^S)\right] = 0, \tag{A.4}$$

[wrt u_t]

$$- \beta^t \Phi l_t^{-\varphi} + \beta^t \lambda_{ct} \bar{b}_t + \beta^t \lambda_{n^M t} \psi_t^{hM} s_t + \beta^t \lambda_{n^S t} \psi_t^{hS} (1 - s_t) = 0, \tag{A.5}$$

[wrt s_t]

$$\lambda_{n_t^M} \psi_t^{hM} - \lambda_{n_t^S} \psi_t^{hS} = 0. \tag{A.6}$$

To simplify the system of equations, we combine (A.1) and (A.2),

$$c_t^{-\eta} = \beta E_t \left[c_{t+1}^{-\eta}(1 - \delta + r_{t+1})\right], \tag{A.7}$$

and then rearrange terms in (A.3) - (A.6),

$$\lambda_{n^M t} = \beta E_t \left[-\Phi l_{t+1}^{-\varphi} + c_{t+1}^{-\eta} w_{t+1}^M + \lambda_{n^M t+1}(1 - \sigma^M)\right], \tag{A.8}$$

$$\lambda_{n^S t} = \beta E_t \left[-\Phi l_{t+1}^{-\varphi} + c_{t+1}^{-\eta} w_{t+1}^S + \lambda_{n^S t+1}(1-\sigma^S) \right], \tag{A.9}$$

$$\Phi l_t^{-\varphi} = \lambda_{n^M t} \psi_t^{hM} s_t + \lambda_{n^S t} \psi_t^{hS}(1-s_t) + \lambda_{ct} \bar{b}_t, \tag{A.10}$$

$$\lambda_{n_t^M} \psi_t^{hM} = \lambda_{n_t^S} \psi_t^{hS}. \tag{A.11}$$

2 The wage bargaining problem

2.1 Manufacturing sector

The maximization problem is written as,

$$\max_{w_t^M} \left\{ \left(1-\vartheta^M\right) \ln V_{n^M t}^h + \vartheta^M \ln V_{n^M t}^f \right\}, \tag{A.12}$$

where

$$V_{n^M t}^h = -\Phi l_t^{-\varphi} + \lambda_{ct} w_t^M + (1-\sigma^M)\lambda_{n^M t}, \tag{A.13}$$

and

$$V_{n^M t}^f = p_t^M (1-\zeta) \left(\frac{M_t}{n_t^M}\right)^{\frac{1}{\alpha}} - w_t^M + \frac{\left(1-\sigma^M\right)\kappa^M}{\psi_t^{fM}}. \tag{A.14}$$

The first-order condition is written as,

$$\vartheta^M V_{n^M t}^h = \left(1-\vartheta^M\right) \lambda_{ct} V_{n^M t}^f.$$

Using (A.13) and (A.14), we obtain,

$$\vartheta^M \left(\lambda_{ct} w_t^M - \Phi l_t^{-\varphi} + (1-\sigma^M)\lambda_{n^M t}\right) = \\ \left(1-\vartheta^M\right) \lambda_{ct} \left(p_t^M (1-\zeta) \left(\frac{M_t}{n_t^M}\right)^{\frac{1}{\alpha}} - w_t^M + \frac{\left(1-\sigma^M\right)\kappa^M}{\psi_t^{fM}} \right),$$

$$\vartheta^M \lambda_{ct} w_t^M + \vartheta^M \left(-\Phi l_t^{-\varphi} + (1-\sigma^M)\lambda_{n^M t}\right) = \\ -(1-\vartheta^M)\lambda_{ct} w_t^M + \left(1-\vartheta^M\right) \lambda_{ct} \left(p_t^M (1-\zeta) \left(\frac{M_t}{n_t^M}\right)^{\frac{1}{\alpha}} + \frac{\left(1-\sigma^M\right)\kappa^M}{\psi_t^{fM}} \right),$$

$$\vartheta^M \lambda_{ct} w_t^M + (1-\vartheta^M)\lambda_{ct} w_t^M =$$

$$-\vartheta^M \left(-\Phi l_t^{-\varphi} + (1-\sigma^M)\lambda_{n^M t}\right) + \left(1-\vartheta^M\right)\lambda_{ct}\left(p_t^M (1-\zeta)\left(\frac{M_t}{n_t^M}\right)^{\frac{1}{\alpha}} + \frac{\left(1-\sigma^M\right)\kappa^M}{\psi_t^{fM}}\right).$$

The final expression for the wage in the manufacturing sector is,

$$w_t^M = \left(1-\vartheta^M\right)\left(p_t^M (1-\zeta)\left(\frac{M_t}{n_t^M}\right)^{\frac{1}{\alpha}} + \frac{\left(1-\sigma^M\right)\kappa^M}{\psi_t^{fM}}\right) - \frac{\vartheta^M}{\lambda_{ct}}\left((1-\sigma^M)\lambda_{n^M t} - \Phi l_t^{-\varphi}\right).$$

2.2 Service sector

Similarly, the maximization problem is written as,

$$\max_{w_t^S}\left\{\left(1-\vartheta^S\right)\ln V_{n^S t}^h + \vartheta^S \ln V_{n^S t}^f\right\}, \tag{A.15}$$

where

$$V_{n^S t}^h = \lambda_{ct} w_t^S - \Phi l_t^{-\varphi} + \left(1-\sigma^S\right)\lambda_{n^S t}, \tag{A.16}$$

and

$$V_{n^S t}^f = p_t^S b \frac{S_t}{n_t^S} - w_t^S + \frac{\left(1-\sigma^S\right)\kappa^S}{\psi_t^{fS}}. \tag{A.17}$$

The first-order condition is written as,

$$\vartheta^S V_{n^S t}^h = \left(1-\vartheta^S\right)\lambda_{ct} V_{n^S t}^f.$$

Using (A.16) and (A.17) we obtain,

$$\vartheta^S(\lambda_{ct} w_t^S - \Phi l_t^{-\varphi} + \left(1-\sigma^S\right)\lambda_{n^S t}) = \left(1-\vartheta^S\right)\lambda_{ct}(p_t^S b \frac{S_t}{n_t^S} - w_t^S + \frac{\left(1-\sigma^S\right)\kappa^S}{\psi_t^{fS}}).$$

We perform a simple algebra below,

$$\vartheta^S \lambda_{ct} w_t^S + \vartheta^S(-\Phi l_t^{-\varphi} + \left(1-\sigma^S\right)\lambda_{n^S t}) = -\left(1-\vartheta^S\right)\lambda_{ct} w_t^S + \left(1-\vartheta^S\right)\lambda_{ct}(p_t^S b \frac{S_t}{n_t^S} + \frac{\left(1-\sigma^S\right)\kappa^S}{\psi_t^{fS}}),$$

$$\vartheta^S \lambda_{ct} w_t^S + \left(1-\vartheta^S\right) \lambda_{ct} w_t^S = \left(1-\vartheta^S\right) \lambda_{ct} (p_t^S b \frac{S_t}{n_t^S} + \frac{\left(1-\sigma^S\right)\kappa^S}{\psi_t^{fS}}) - \vartheta^S (-\Phi l_t^{-\varphi} + \left(1-\sigma^S\right) \lambda_{n^{S_t}}),$$

$$w_t^S \lambda_{ct} = \left(1-\vartheta^S\right) \lambda_{ct} (p_t^S b \frac{S_t}{n_t^S} + \frac{\left(1-\sigma^S\right)\kappa^S}{\psi_t^{fS}}) - \vartheta^S (-\Phi l_t^{-\varphi} + \left(1-\sigma^S\right) \lambda_{n^{S_t}}).$$

The final expression for the equilibrium wage is given by,

$$w_t^S = \left(1-\vartheta^S\right) \left(p_t^S b \frac{S_t}{n_t^S} + \frac{\left(1-\sigma^S\right)\kappa^S}{\psi_t^{fS}} \right) - \frac{\vartheta^S}{\lambda_{ct}} (\left(1-\sigma^S\right) \lambda_{n^{S_t}} - \Phi l_t^{-\varphi}).$$

3 Resource constraint

From the household budget constraint,

$$c_t + i_t = r_t k_t + w_t^M n_t^M + w_t^S n_t^S + \bar{b}_t u_t - T_t + \Pi_t^M + \Pi_t^S.$$

We substitute the expressions of the instantaneous profits of firms,

$$\Pi_t^M = p_t^M M_t - w_t^M n_t^M - r_t k_t - \kappa^M v_t^M,$$

$$\Pi_t^S = p_t^S S_t - w_t^S n_t^S - \kappa^S v_t^S,$$

and obtain,

$$c_t + i_t = p_t^M M_t + p_t^S S_t - \kappa^S v_t^S - \kappa^M v_t^m.$$

Because of the constant returns to scale and frictionless production of the final good, we have,

$$Y_t = p_t^M M_t + p_t^S S_t.$$

So, we obtain the resource constraint,

$$Y_t = c_t + i_t + \kappa^S v_t^S + \kappa^M v_t^m.$$

4 Model Solution

We report below all equations that need to be satisfied in equilibrium.

Labor markets

$$m_t^M = \mu_1 (\upsilon_t^M)^{\mu_2} (u_t^M)^{1-\mu_2} \tag{E.1}$$

$$m_t^S = \mu_1 (\upsilon_t^S)^{\mu_2} (u_t^S)^{1-\mu_2} \tag{E.2}$$

$$n_{t+1}^M = (1-\sigma^M) n_t^M + m_t^M \tag{E.3}$$

$$n_{t+1}^S = (1-\sigma^S) n_t^S + m_t^S \tag{E.4}$$

$$\psi_t^{hM} \equiv \frac{m_t^M}{u_t^M} \tag{E.5}$$

$$\psi_t^{fM} \equiv \frac{m_t^M}{\upsilon_t^M} \tag{E.6}$$

$$\psi_t^{hS} \equiv \frac{m_t^S}{u_t^S} \tag{E.7}$$

$$\psi_t^{fS} \equiv \frac{m_t^S}{\upsilon_t^S} \tag{E.8}$$

Household

$$c_t^{-\eta} = \beta E_t \left[c_{t+1}^{-\eta} (1 - \delta + r_{t+1}) \right] \tag{E.9}$$

$$\lambda_{n^M t} = \beta E_t \left[-\Phi l_{t+1}^{-\varphi} + c_{t+1}^{-\eta} w_{t+1}^M + \lambda_{n^M t+1} (1-\sigma^M) \right] \tag{E.10}$$

$$\lambda_{n^S t} = \beta E_t \left[-\Phi l_{t+1}^{-\varphi} + c_{t+1}^{-\eta} w_{t+1}^S + \lambda_{n^S t+1} (1-\sigma^S) \right] \tag{E.11}$$

$$\Phi l_t^{-\varphi} = \lambda_{n^M t} \psi_t^{hM} s_t + \lambda_{n^S t} \psi_t^{hS} (1 - s_t) + \lambda_{ct} \bar{b}_t \tag{E.12}$$

$$\lambda_{n_t^M} \psi_t^{hM} = \lambda_{n_t^S} \psi_t^{hS} \tag{E.13}$$

$$n_t^M + n_t^S + u_t + l_t = 1 \tag{E.14}$$

$$u_t^M = s_t u_t \tag{E.15}$$

$$u_t^S = (1 - s_t) u_t \tag{E.16}$$

$$c_t + k_{t+1} = (1 - \delta + r_t) k_t + w_t^M n_t^M + w_t^S n_t^S + \bar{b}_t u_t - T_t + \Pi_t^M + \Pi_t^S \tag{E.17}$$

Firms

$$\Lambda_{t,t+1} = \beta \left(\frac{\lambda_{n^{c_t+1}}}{\lambda_{n^{c_t}}} \right)^{-\eta} \tag{E.18}$$

$$\frac{\kappa^M}{\psi_t^{fM}} = E_t \Lambda_{t,t+1} \left[p_{t+1}^M (1 - \zeta) \left(\frac{M_{t+1}}{n_{t+1}^M} \right)^{\frac{1}{\alpha}} - w_{t+1}^M + \frac{\left(1 - \sigma^M\right) \kappa^M}{\psi_{t+1}^{fM}} \right] \tag{E.19}$$

$$r_t = p_t^M \cdot \zeta \left(\frac{M_t}{k_t} \right)^{\frac{1}{\alpha}} \tag{E.20}$$

$$p_t^M = \gamma \left(\frac{Y_t}{M_t} \right)^{\frac{1}{\chi}} \tag{E.21}$$

$$M_t = \left[\zeta k_t^{\frac{\alpha-1}{\alpha}} + (1 - \zeta)(n_t^M)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}}, \tag{E.22}$$

$$Y_t = \left[\gamma M_t^{\frac{\chi-1}{\chi}} + (1 - \gamma) S_t^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}}, \tag{E.23}$$

$$\frac{\kappa^S}{\psi_t^{fS}} = E_t \Lambda_{t,t+1} \left[p_{t+1}^S b \frac{S_{t+1}}{n_{t+1}^S} - w_{t+1}^S + \frac{\left(1 - \sigma^S\right) \kappa^S}{\psi_{t+1}^{fS}} \right] \tag{E.24}$$

$$p_t^S = (1 - \gamma) \left(\frac{Y_t}{S_t} \right)^{\frac{1}{\chi}} \tag{E.25}$$

$$S_t = B (n_t^S)^b \tag{E.26}$$

Wages

$$w_t^S = \left(1-\vartheta^S\right)\left(p_t^S b \frac{S_t}{n_t^S} + \frac{\left(1-\sigma^S\right)\kappa^S}{\psi_t^{fS}}\right) + \frac{\vartheta^S}{\lambda_{ct}}(\Phi l_t^{-\varphi} - \left(1-\sigma^S\right)\lambda_{n^{S_t}}) \tag{E.27}$$

$$w_t^M = \left(1-\vartheta^M\right)\left(p_t^M(1-\zeta)\left(\frac{M_t}{n_t^M}\right)^{\frac{1}{\alpha}} + \frac{\left(1-\sigma^M\right)\kappa^M}{\psi_t^{fM}}\right) + \frac{\vartheta^M}{\lambda_{ct}}(\Phi l_t^{-\varphi} - \left(1-\sigma^M\right)\lambda_{n^{M_t}}) \tag{E.28}$$

5 Steady-state system of equations

$$m^M = \mu_1(\upsilon^M)^{\mu_2}(u^M)^{1-\mu_2} \tag{S.1}$$

$$m^S = \mu_1(\upsilon^S)^{\mu_2}(u^S)^{1-\mu_2} \tag{S.2}$$

$$n^M = \frac{m^M}{\sigma^M} \tag{S.3}$$

$$n^S = \frac{m^S}{\sigma^S} \tag{S.4}$$

$$\psi^{hM} \equiv \frac{m^M}{u^M}, \tag{S.5}$$

$$\psi^{fM} \equiv \frac{m^M}{\upsilon^M}, \tag{S.6}$$

$$\psi^{hS} \equiv \frac{m^S}{u^S}, \tag{S.7}$$

$$\psi^{fS} \equiv \frac{m^S}{\upsilon^S} \tag{S.8}$$

Household

$$r = \frac{1-\beta}{\beta} + \delta \tag{S.9}$$

$$\lambda_{n^M} = \beta\left[-\Phi l^{-\varphi} + c^{-\eta}w^M + \lambda_{n^M}(1-\sigma^M)\right] \tag{S.10}$$

$$\lambda_{n^S} = \beta\left[-\Phi l^{-\varphi} + c^{-\eta}w^S + \lambda_{n^S}(1-\sigma^S)\right] \tag{S.11}$$

$$\Phi l^{-\varphi} = \lambda_{n^M} \psi^{hM} s + \lambda_{n^S} \psi^{hS} (1-s) + \lambda_c \bar{b} \tag{S.12}$$

$$\lambda_{n^M} \psi^{hM} = \lambda_{n^S} \psi^{hS} \tag{S.13}$$

$$n^M + n^S + u + l = 1 \tag{S.14}$$

$$u^M = su \tag{S.15}$$

$$u^S = (1-s)u \tag{S.16}$$

$$c = k(r-\delta) + w^M n^M + w^S n^S + \bar{b}u \tag{S.17}$$

Firms

$$\frac{\kappa^S}{\psi^{fS}} = \beta \left[p^S b \frac{S}{n^S} - w^S + \frac{\left(1-\sigma^S\right) \kappa^S}{\psi^{fS}} \right] \tag{S.18}$$

$$p^S = (1-\gamma) \left(\frac{Y}{S} \right)^{\frac{1}{\chi}} \tag{S.19}$$

$$S = B(n^S)^b \tag{S.20}$$

$$\frac{\kappa^M}{\psi_t^{fM}} = \beta \left[p^M (1-\zeta) \left(\frac{M}{n^M} \right)^{\frac{1}{\alpha}} - w^M + \frac{\left(1-\sigma^M\right) \kappa^M}{\psi^{fM}} \right] \tag{S.21}$$

$$r = p^M \cdot \zeta \left(\frac{M}{k} \right)^{\frac{1}{\alpha}} \tag{S.22}$$

$$p^M = \gamma \left(\frac{Y}{M} \right)^{\frac{1}{\chi}} \tag{S.23}$$

$$M = \left[\zeta k^{\frac{\alpha-1}{\alpha}} + (1-\zeta)(n^M)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}}, \tag{S.24}$$

Wages

$$w^S = \left(1-\vartheta^S\right)\left(p^S b \frac{S}{n^S} + \frac{\left(1-\sigma^S\right)\kappa^S}{\psi^{fS}}\right) + \frac{\vartheta^S}{\lambda_c}(\Phi l^{-\varphi} - \left(1-\sigma^S\right)\lambda_{n^S}) \tag{S.25}$$

$$w^M = \left(1-\vartheta^M\right)\left(p^M(1-\zeta)\left(\frac{M}{n^M}\right)^{\frac{1}{\alpha}} + \frac{\left(1-\sigma^M\right)\kappa^M}{\psi^{fM}}\right) + \frac{\vartheta^M}{\lambda_c}(\Phi l^{-\varphi} - \left(1-\sigma^M\right)\lambda_{n^M}) \tag{S.26}$$

5.1 Steady state - simplified system

Given the unknowns $u, \upsilon^M, \upsilon^S, k, s$, the following variables are determined sequentially:

$$r = \frac{1-\beta}{\beta} + \delta \tag{SS.1}$$

$$u^M = su \tag{SS.2}$$

$$u^S = (1-s)u \tag{SS.3}$$

$$m^M = \mu_1(\upsilon^M)^{\mu_2}(u^M)^{1-\mu_2} \tag{SS.4}$$

$$m^S = \mu_1(\upsilon^S)^{\mu_2}(u^S)^{1-\mu_2} \tag{SS.5}$$

$$n^M = \frac{m^M}{\sigma^M} \tag{SS.6}$$

$$n^S = \frac{m^S}{\sigma^S} \tag{SS.7}$$

$$\psi^{hM} \equiv \frac{m^M}{u^M}, \tag{SS.8}$$

$$\psi^{fM} \equiv \frac{m^M}{\upsilon^M}, \tag{SS.9}$$

$$\psi^{hS} \equiv \frac{m^S}{u^S}, \tag{SS.10}$$

$$\psi^{fS} \equiv \frac{m^S}{\upsilon^S} \tag{SS.11}$$

$$l = 1 - u - n^M - n^S \tag{SS.12}$$

$$S = B(n^S)^b \tag{SS.13}$$

$$M = \left[\zeta k^{\frac{\alpha-1}{\alpha}} + (1-\zeta)(n^M)^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}}, \tag{SS.14}$$

$$Y = \left[\gamma M^{\frac{\chi-1}{\chi}} + (1-\gamma)S^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}, \tag{SS.15}$$

$$p^S = (1-\gamma)\left(\frac{Y}{S}\right)^{\frac{1}{\chi}} \tag{SS.16}$$

$$p^M = \gamma\left(\frac{Y}{M}\right)^{\frac{1}{\chi}} \tag{SS.17}$$

The wages of two sectors are obtained from the inverted FOCs with respect to sectoral employment:

$$w^S = p^S b\frac{S}{n^S} - \frac{\kappa^S}{\psi^{fS}\beta} + \frac{(1-\sigma^S)\kappa^S}{\psi^{fS}} \tag{SS.18}$$

$$w^M = p^M(1-\zeta)\left(\frac{M}{n^M}\right)^{\frac{1}{\alpha}} - \frac{\kappa^M}{\psi^{fM}\beta} + \frac{\left(1-\sigma^M\right)\kappa^M}{\psi^{fM}} \tag{SS.19}$$

$$\bar{b} = \varpi\frac{(w^m n^M + w^S n^S)}{n^M + n^S} \tag{SS.20}$$

$$T = \bar{b}u \tag{SS.21}$$

$$c = k(r-\delta) + w^M n^M + w^S n^S + \bar{b}u - T + \Pi^M + \Pi^S \tag{SS.22}$$

$$\lambda_c = c^{-\eta} \tag{SS.23}$$

$$\lambda_{n^M} = \frac{\beta}{1-\beta(1-\sigma)}\left(c^{-\eta}w^M - \Phi l^{-\varphi}\right) \tag{SS.24}$$

$$\lambda_{n^S} = \frac{\beta}{1-\beta(1-\sigma)} \left(c^{-\eta} w^S - \Phi l^{-\varphi} \right) \tag{SS.25}$$

To solve for the unknowns, we numerically find the solution to the following system of equations, in which we use expressions (S.1) to (S.25).

$$\Phi l^{-\varphi} = \lambda_{n^M} \psi^{hM} s + \lambda_{n^S} \psi^{hS} (1-s) + \lambda_c \bar{b} \tag{SS.26}$$

$$p^M \cdot \zeta \left(\frac{M}{k} \right)^{1-\alpha} - r = 0 \tag{SS.27}$$

$$\frac{\kappa^S}{\psi^{fS}} = \beta \left[p^S b \frac{S}{n^S} - w^S + \frac{\left(1-\sigma^S\right) \kappa^S}{\psi^{fS}} \right] \tag{SS.28}$$

$$\frac{\kappa^M}{\psi_t^{fM}} = \beta \left[p^M (1-\zeta) \left(\frac{M}{n^M} \right)^{1-\alpha} - w^M + \frac{\left(1-\sigma^M\right) \kappa^M}{\psi^{fM}} \right] \tag{SS.29}$$

$$w^S = \left(1-\vartheta^S\right) \left(p^S b \frac{S}{n^S} + \frac{\left(1-\sigma^S\right) \kappa^S}{\psi^{fS}} \right) + \frac{\vartheta^S}{\lambda_c} (\Phi l^{-\varphi} - \left(1-\sigma^S\right) \lambda_{n^S}) \tag{SS.30}$$

$$w^M = \left(1-\vartheta^M\right) \left(p^M (1-\zeta) \left(\frac{M}{n^M} \right)^{1-\alpha} + \frac{\left(1-\sigma^M\right) \kappa^M}{\psi^{fM}} \right) + \frac{\vartheta^M}{\lambda_c} (\Phi l^{-\varphi} - \left(1-\sigma^M\right) \lambda_{n^M}) \tag{SS.31}$$

$$\lambda_{n^M} \psi^{hM} = \lambda_{n^S} \psi^{hS} \tag{SS.32}$$

6 The Role of the Frisch Elasticity of Labor Supply

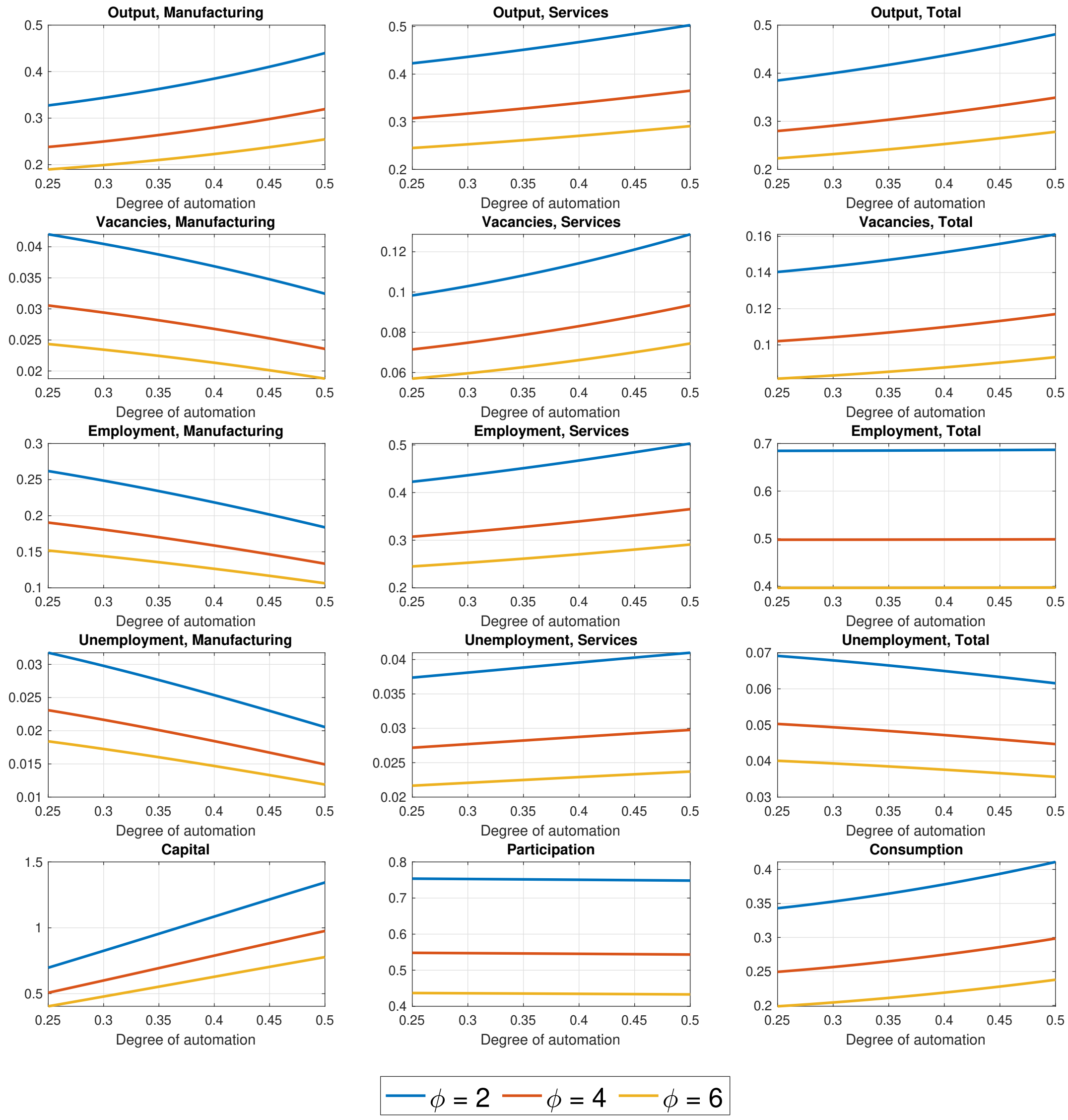

Figure A.1: Steady-state effects of automation in a two-sector economy: Different values of the Frisch elasticity

Note: The y-axis shows steady-state levels. The blue line refers to the baseline calibration ($\phi = 2$).

7 The Role of the Elasticities of Substitution

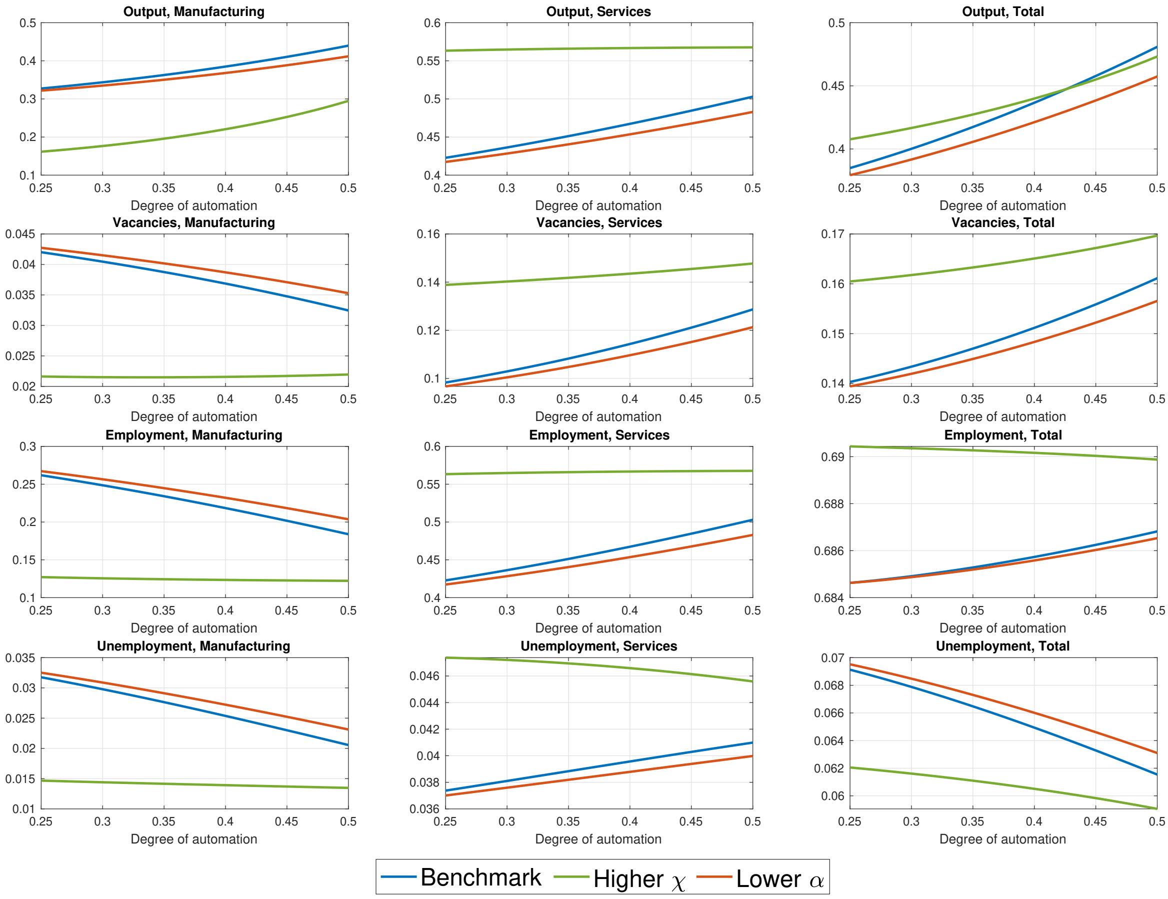

Figure A.2: Steady-state effects of automation in a two-sector economy: Different elasticities of substitution between capital and labor ($\alpha = 0.7$) and between the two goods ($\chi = 1.5$)

Note: The y-axis shows steady-state levels.

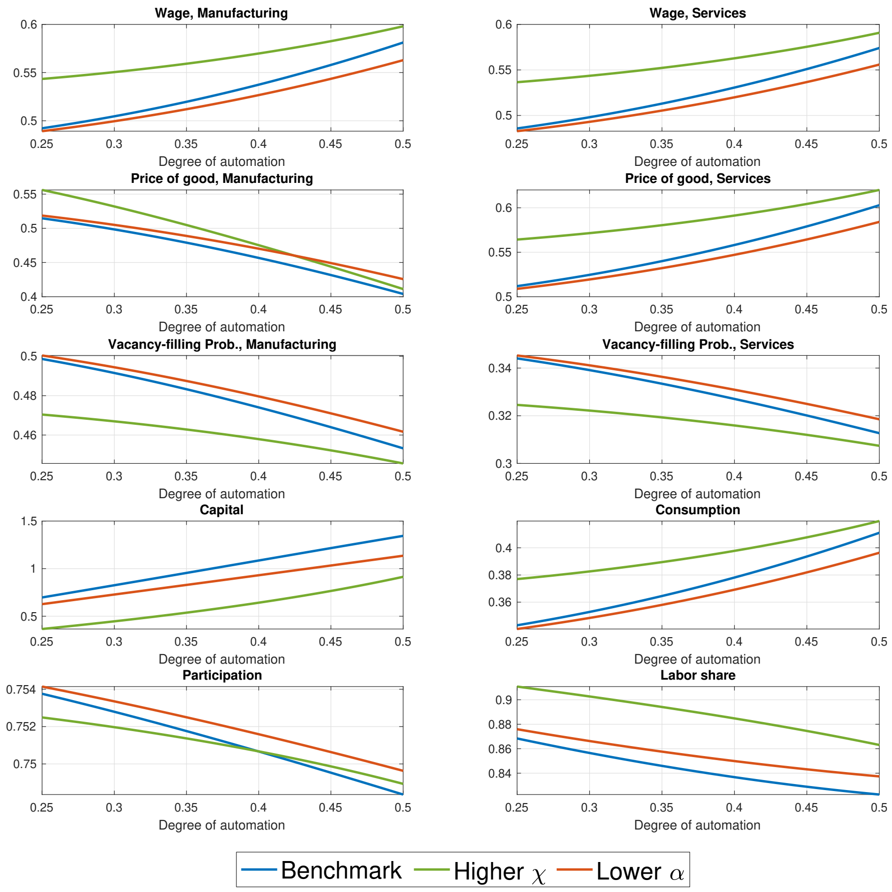

Figure A.3: Steady-state effects of automation in a two-sector economy: Different elasticities of substitution between capital and labor ($\alpha = 0.7$) and between the two goods ($\chi = 1.5$) (continued)

Note: The y-axis shows steady-state levels.

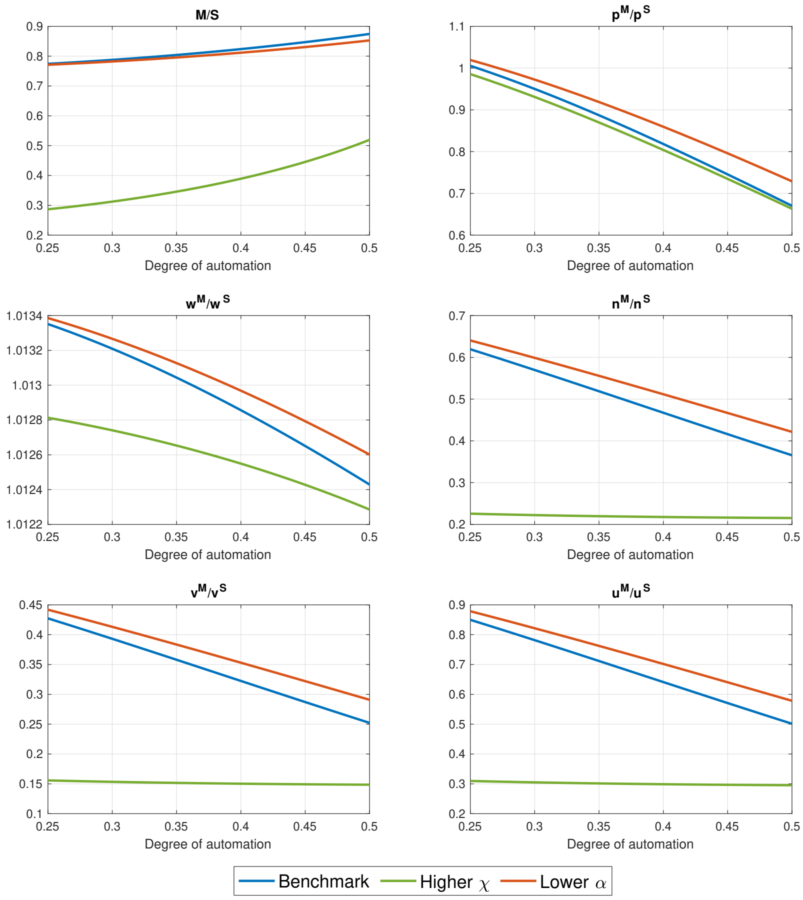

Figure A.4: Steady-state effects of automation on key ratios in a two-sector economy: Different elasticities of substitution between capital and labor ($\alpha = 0.7$) and between the two goods ($\chi = 1.5$)

Note: The y-axis shows steady-state levels.