Automation and Sectoral Reallocation[*]

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Abstract

Empirical evidence in Dauth et al. (2021) suggests that industrial robot adoption in Germany has led to a sectoral reallocation of employment from manufacturing to services, leaving total employment unaffected. We rationalize this evidence through the lens of a general equilibrium model with two sectors, matching frictions, and endogenous participation. Automation induces firms to create fewer vacancies and job seekers to search less in the automatable sector (manufacturing). The service sector experiences a positive spillover effect due to the sectoral complementarity in the production of the final good and the positive income effect for the household. Analysis across steady states shows that the reduction in manufacturing employment can be offset by the increase in service employment. The model can also replicate the magnitude of the decline in the ratio of manufacturing employment to service employment in Germany from 1994 to 2014.

JEL classification codes: E24, O14, O33, J22, J23.

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1 Introduction

As a result of improved capabilities and falling production costs, the global operational stock of industrial robots rose by about 65% within five years (2013-2018). The Covid-19 crisis is expected to accelerate further the speed of automation (see, e.g., Dolado et al. (2020a) and Leduc and Liu (2020a)). In addition to the significant implications for labor markets, recent evidence reveals that higher exposure to robot adoption has increased support for nationalist and radical-right parties in Western Europe (Anelli et al. (2020)).

Academic and policy debates have focused on whether robots cause job displacement or job creation in the economy. On the one hand, a negative displacement effect arises from the fact that robots can outperform workers in some tasks. For instance, Acemoglu and Restrepo (2020) recently find that each robot installed in the US replaces six workers. On the other hand, a positive productivity effect occurs because machines can help fewer workers produce more output, which increases labor demand. In this vein, the seminal work by Graetz and Michaels (2018) finds, using industry-level data from 17 countries, that cumulative changes in robot adoption from 1993 to 2007 boost labor productivity and raise wages.[1]

Notably, the adjustment in other parts of the economy and the potential sector spillover effects – for instance, when other sectors expand to absorb the labor freed from robot adoption – have received little attention so far. According to empirical evidence from Germany in Dauth et al. (2021), industrial robots have changed the composition but not the aggregate size of employment, with job gains in services offsetting the negative impact on manufacturing employment. Figure 1 shows the evolution of employment and employees' compensation (as a share of GDP) in the two sectors along with the stock of industrial robots in the country with the highest robot density in Europe (see Figure 2).

To rationalize the empirical evidence on the automation-driven sectoral reallocation of labor in Germany, we develop a general equilibrium model with two production sectors, a labor market participation choice, and matching frictions.[2] Our modeling framework for automation (see Section 2) is consistent with the microfoundations derived by Acemoglu and Restrepo (2018) and close in spirit to Bergholt et al. (2020).[3] The presence of unemployment in the model is crucial as we seek to

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[1] There are two main strands in the literature regarding a tangible measure of automation: information-and-communication-technology capital (see, e.g., Eden and Gaggl (2018)) and robotics (see, e.g., Graetz and Michaels (2018)).

[2] For empirical work on the decline in manufacturing and the rise in services, see a novel dataset for 10 sectors, 23 countries, and 150 years compiled by Priftis and Shakhnov (2020).

[3] Note that Bergholt et al. (2020) examine impulse responses from a New Keynesian model, while
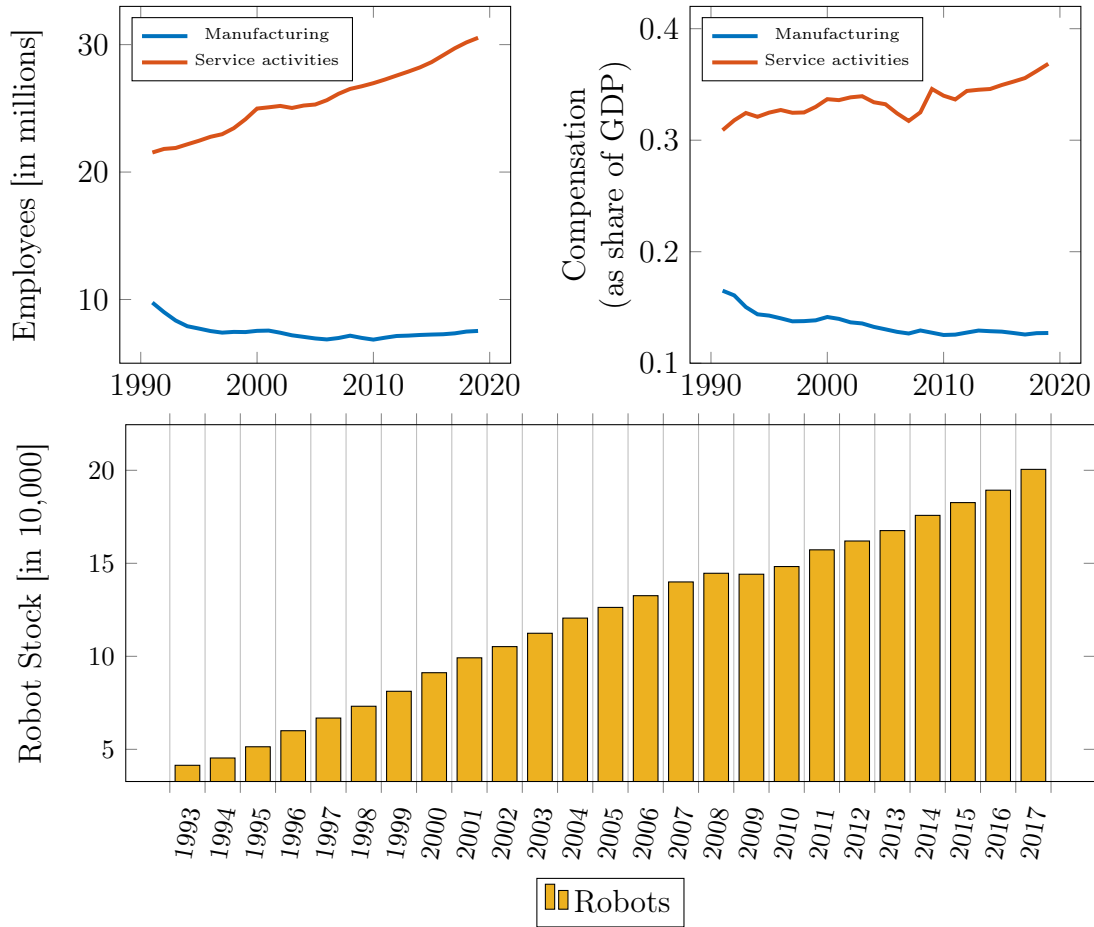
Figure 1: Industrial robots, employment and employees' compensation in Germany

Note: Numbers of employees and the levels of their compensation (as share of GDP) in the manufacturing and service sectors come from the Federal Statistical Office (Destatis). Data on the stock of industrial robots comes from the International Federation of Robotics (IFR).

explain how total employment can remain constant when labor reallocates between the two sectors. Without unemployment, that would be true by construction, while it is a result of our model. Furthermore, the inclusion of labor market frictions allows us to study the impact of automation on endogenous job creation. The presence of the extensive margin in our model is motivated by recent literature highlighting the negative effect of automation on participation (see, e.g., Lerch (2020), Grigoli et al. (2020), Jaimovich et al. (2020), and Lerch (2020)). Overall, the adjustment of sectoral labor markets in response to automation takes place in the model through three channels: (i) job creation, (ii) sector-specific search of unemployed job seekers, and (iii) participation. Since our representative household model is capable of rationalizing the empirical evidence mentioned above, we abstract from heterogeneous households for simplicity.

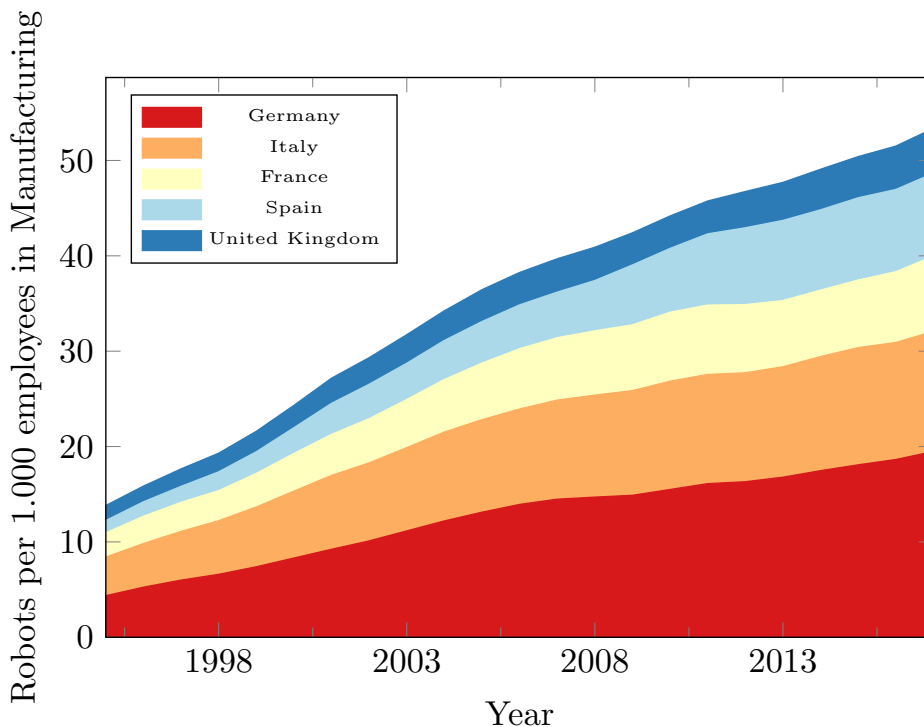we focus on long-run effects through analysis across steady states.

Figure 2: Industrial robot density in the manufacturing sector of European economies

Note: Data on the stock of industrial robots comes from the International Federation of Robotics (IFR). We define the manufacturing sector as the aggregate of Industries A-F in the German WZ08 industry classification.

Our main findings can be summarized as follows. In the model, as in the empirics of Dauth et al. (2021), automation induces firms to create fewer vacancies and job seekers to search less in the robot-exposed sector (manufacturing). The model is able to replicate the empirical evolution of employment and employees' compensation in manufacturing and services (Figure 1). The service sector experiences a positive spillover effect and expands. Labor demand in services increases since the two sectoral goods are gross complements in the production of the final consumption good. This result is consistent with the model of Acemoglu and Restrepo (2020), where higher robot adoption increases demand for complementary inputs. Additionally, as income rises, consumption demand increases (positive income effect), also contributing to the spillover effect. Calibrating the model for Germany, we show through analysis across steady states that the reduction in manufacturing employment can be offset by the increase in service employment, thus leaving aggregate employment mostly unaffected.

Our analysis highlights vacancy creation (labor demand) as the primary channel through which the two labor markets adjust to automation. The elasticities of substitution between capital and labor in manufacturing production and between

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automatable (manufacturing) and non-automatable (service) goods play an important role in the sectoral reallocation of labor, while the sectoral mobility of labor and the positive income effect also matter. Furthermore, the model generates a negative effect of automation on labor market participation in line with the literature, but, overall, results do not depend crucially on the extensive margin.

The model can replicate the magnitude of the decline in the ratio of manufacturing employment to service employment in Germany from 1994 to 2014. Specifically, we take from the German data the values of the capital share in manufacturing in these two years. Then, we compute the values of the degree of automation in our model that generate these two values in the corresponding steady states, keeping the rest of the calibration unchanged. We find that in the second steady state (for 2014) the model predicts a decline of 27% in the ratio of manufacturing employment to service employment, which is close to the one found in the data (32%).

Related Literature. Abstracting from labor market frictions, Bergholt et al. (2020) examine impulse responses to an automation shock, modeled as an exogenous increase in the weight of capital in the production function of a New Keynesian model. They find that, among four possible explanations, automation is the main driver of the long-run labor share. In macroeconomic models with labor frictions, the role of automation remains little explored. Leduc and Liu (2020b) provide the first quantitative general equilibrium evaluation of the interaction between automation and labor market fluctuations over the business cycle. Automation acts as an endogenous wage rigidity by posing a threat to workers in wage negotiations. Leduc and Liu (2020a) extend the previous Real Business Cycle model with nominal rigidities. They find that pandemic-induced uncertainty shocks to worker productivity stimulate automation, which helps mitigate the negative impact on aggregate demand. Models with automation, heterogeneous households, and matching frictions are developed by Cords and Prettner (2019) and Jaimovich et al. (2020) to study the impact on inequality.

Very few studies in the automation literature have considered a two-sector economy without accounting for labor market frictions. Focusing on inequality, Berg et al. (2018) show that the inclusion of a non-automation sector amplifies the high-skill labor gains and low-skill labor losses from automation. A non-automatable sector is included in an overlapping generations setting by Sachs et al. (2019). The study shows how short-term increases in consumption enabled by robots may lead to long-term immiseration and how government intervention can take place. To the best of our knowledge, we are the first to build a two-sector general equilib-

rium model with search and matching frictions to analyze the long-run impact of automation on both sectoral and aggregate employment.

Structure. Section 2 lays out the model. Section 3 establishes the equilibrium relationship between relative labor demand and labor supply in the two-sector economy. Section 4 discusses the calibration strategy. Section 5 presents the results. Section 6 investigates the role of key parameters and features of the model. Section 7 concludes.

2 The Model

We construct a general equilibrium model featuring search and matching frictions, endogenous labor decisions, and two sectors (manufacturing and services). Figure 3 provides an overview of the model.

On the production side, there is a representative firm in each of the two sectors. Manufacturing output is produced with capital and labor as inputs, while output in services is produced with labor only. The outputs of the two sectors are costlessly aggregated into the final consumption good.

On the household side, there is a representative household consisting of employees, unemployed job seekers, and labor force nonparticipants. The household rents out its capital to the manufacturing firm, purchases the final consumption good, and receives dividends through owning the two firms.

2.1 Labor markets

Jobs are created through a matching function. For $j = M, S$ denoting the manufacturing and service sectors, let v_t^j be the number of vacancies and u_t^j the number of job seekers. We assume matching functions of the form,

$$m_t^j = \mu_1^j (v_t^j)^{\mu_2} (u_t^j)^{1-\mu_2}, \tag{1}$$

where the efficiency of the matching process is μ_1^j and μ_2^j denotes the elasticity of matches with respect to vacancies. For each sector, we define the hiring probability ψ_t^{hj} and the vacancy-filling probability ψ_t^{fj},

$$\psi_t^{hj} \equiv \frac{m_t^j}{u_t^j}, \qquad \psi_t^{fj} \equiv \frac{m_t^j}{v_t^j}.$$
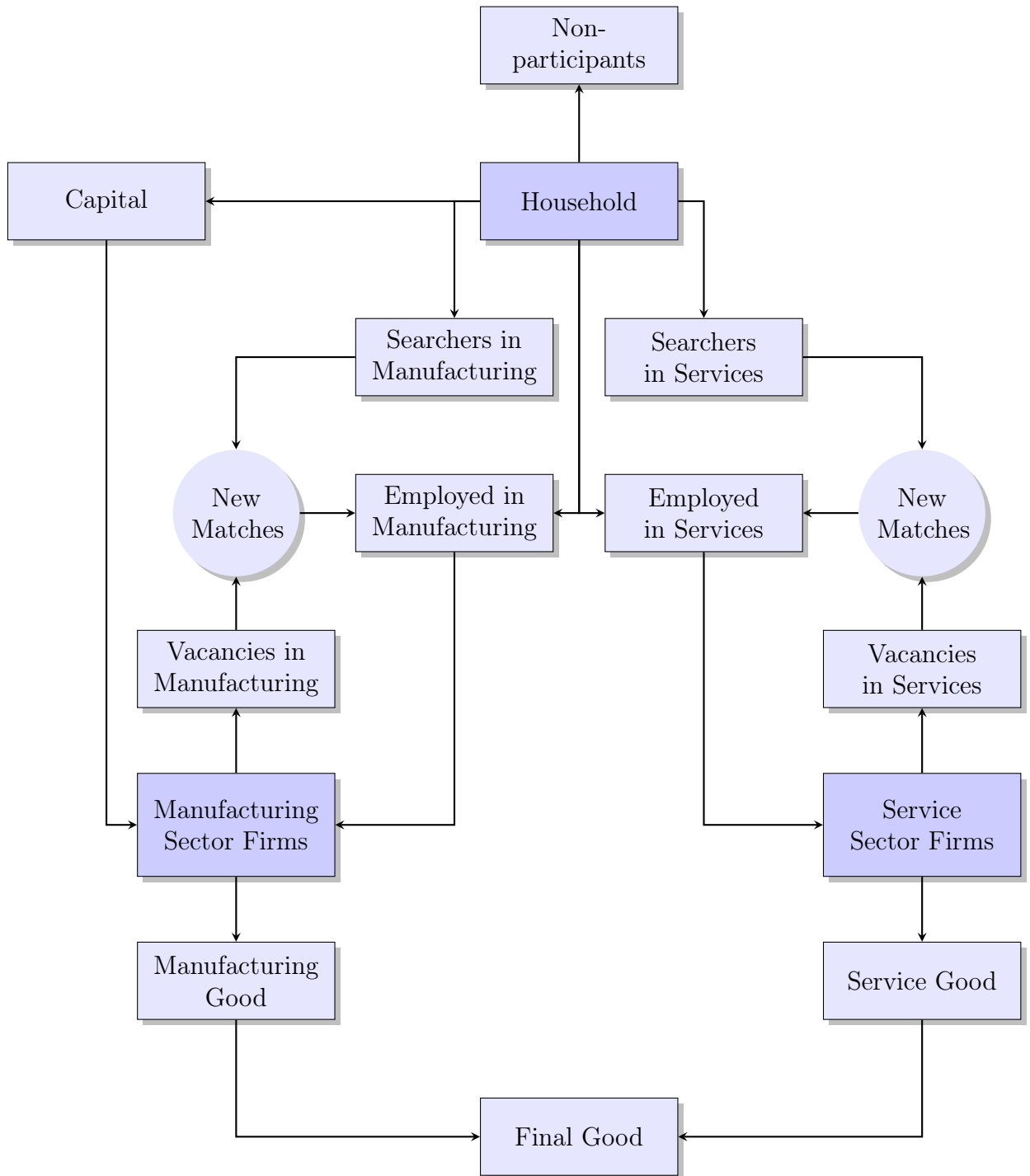
Figure 3: Model overview

Labor market tightness $\theta_t^j \equiv v_t^j / u_t^j$ determines the matching market prospects of firms and workers. The probability that a worker finds a vacancy is an increasing function of labor market tightness, $\psi_t^{hj} = f(\theta_t^j)$, while the probability that a job vacancy is matched with an unemployed worker is a decreasing function of tightness, $\psi_t^{fj} = f(\theta_t^j)/\theta_t^j$.

In each period, jobs are destroyed at a constant fraction σ^j and m_t^j new matches are formed. The law of motion of employment n_t^j is then given by,

$$n_{t+1}^j = (1 - \sigma^j)n_t^j + m_t^j = (1 - \sigma^j)n_t^j + \psi_t^{hj}u_t^j. \tag{2}$$

Using the vacancy-filling probability, we obtain an equivalent expression,

$$n_{t+1}^j = (1 - \sigma^j)n_t^j + \psi_t^{fj}v_t^j. \tag{3}$$

2.2 Household

Next, we present the structure of the household side in the model and the corresponding optimization problem.

2.2.1 Utility function and budget constraint

The representative household consists of a continuum of infinitely lived members. Utility is derived from consumption c_t and from leisure, which corresponds to the fraction of members out of the labor force l_t. The instantaneous utility function is given by,

$$U(c_t, l_t) = \frac{c_t^{1-\eta}}{1 - \eta} + \Phi \frac{l_t^{1-\varphi}}{1 - \varphi},$$

where η is the inverse of the intertemporal elasticity of substitution, $\Phi > 0$ is the relative preference for leisure and φ is the inverse of the Frisch elasticity of labor supply. At any point in time, a fraction n_t^M (n_t^S) of the household's members are employees in the manufacturing (service) sector. The household chooses the fraction of the unemployed actively searching for a job u_t versus those who are out of the labor force enjoying leisure l_t so that

$$n_t^M + n_t^S + u_t + l_t = 1. \tag{4}$$

Of the unemployed u_t, the household chooses the fraction of job seekers who look for a job in the manufacturing sector s_t while the remaining $1 - s_t$ search in services,

so that

$$u_t = s_t u_t + (1 - s_t)u_t = u_t^M + u_t^S, \tag{5}$$

where $u_t^M \equiv s_t u_t$ and $u_t^S \equiv (1 - s_t)u_t$. The household accumulates assets, evolving over time according to

$$k_{t+1} = i_t + (1 - \delta)k_t, \tag{6}$$

where i_t is investment and δ is a constant depreciation rate. The household budget constraint is given by,

$$c_t + i_t \leq r_t k_t + w_t^M n_t^M + w_t^S n_t^S + \bar{b}_t u_t - T_t + \Pi_t^M + \Pi_t^S, \tag{7}$$

where w_t^j is the real wage in each sector, r_t is the real return on assets, $\bar{b}_t$ is the unemployment benefit (see Section 4), T_t refers to lump-sum taxes that adjust to satisfy the government budget, i.e. $\bar{b}_t u_t = T_t$, and Π_t^j for $j = M, S$ denotes dividends received from ownership of the firms. We model the unemployment benefit as a share ϖ of the average wage in the economy through the function $\bar{b}_t = \varpi \frac{(w_t^M n_t^M + w_t^S n_t^S)}{n_t^M + n_t^S}$.

2.2.2 The optimization problem

The household maximizes the expected lifetime utility subject to equations (1), (2), (4), (5), (6), and (7) (for details, see the Online Appendix). Denoting by $\lambda_{n^M t}$, $\lambda_{n^S t}$, and λ_{ct} the Lagrange multipliers on equations (2) for $j = S, M$ and (7), the first-order conditions with respect to c_t, k_{t+1}, n_{t+1}^M, n_{t+1}^S, u_t and s_t are given by,

$$c_t^{-\eta} = \lambda_{ct}, \tag{8}$$

$$\lambda_{ct} = \beta E_t \left[\lambda_{ct+1}(1 - \delta + r_{t+1}) \right], \tag{9}$$

$$\lambda_{n^M t} = \beta E_t \left[-\Phi l_{t+1}^{-\varphi} + c_{t+1}^{-\eta} w_{t+1}^M + \lambda_{n^M t+1}(1 - \sigma^M) \right], \tag{10}$$

$$\lambda_{n^S t} = \beta E_t \left[-\Phi l_{t+1}^{-\varphi} + c_{t+1}^{-\eta} w_{t+1}^S + \lambda_{n^S t+1}(1 - \sigma^S) \right], \tag{11}$$

$$\Phi l_t^{-\varphi} - \lambda_{n^M t} \psi_t^{hM} s_t - \lambda_{n^S t} \psi_t^{hS}(1 - s_t) = \lambda_{ct} \bar{b}_t, \tag{12}$$

$$\lambda_{n_t^M} \psi_t^{hM} = \lambda_{n_t^S} \psi_t^{hS}. \tag{13}$$

Equations (8) and (9) are the non-arbitrage conditions for the returns to consumption and capital. Equations (10) and (11) relate the expected marginal value of being employed in each sector to the utility loss from the reduction in leisure, the wage, and the continuation value, which depends on the separation probability. Equation (12) states that the value of being unemployed (rather than enjoying leisure) should equal the marginal utility from leisure minus the expected marginal values of being employed in each sector, weighted by the respective job finding probabilities and shares of job seekers. Equation (13) states the choice of the share s_t is such that the expected marginal values of being employed, weighted by the job finding probabilities, are equal in the two sectors. Notice that the marginal value to the household of an additional member employed in each sector is given by,

$$V_{n^M_t}^h = -\Phi l_t^{-\varphi} + \lambda_{ct} w_t^M + (1 - \sigma^M)\lambda_{n^M t}, \tag{14}$$

$$V_{n^S_t}^h = -\Phi l_t^{-\varphi} + \lambda_{ct} w_t^S + (1 - \sigma^S)\lambda_{n^S t}. \tag{15}$$

2.3 Production

We now turn to the structure of the production side in the economy and present the optimization problem of the firms in the two sectors.

2.3.1 Final good

There are three goods produced in the economy. These include two intermediate goods, namely manufacturing and service goods (M_t and S_t), which are combined in the production of the final good Y_t according to a CES technology,

$$Y_t = \left[\gamma M_t^{\frac{\chi-1}{\chi}} + (1 - \gamma)S_t^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}, \tag{16}$$

where $0 < \gamma < 1$ denotes the weight attached to the manufacturing good versus the service good and χ is the elasticity of substitution.

The three goods are sold in competitive markets and we assume that the final good is the numeraire. Therefore, the prices of the sectoral goods equal the marginal products,

$$p_t^M = \frac{\partial Y_t}{\partial M_t} = \gamma \left(\frac{Y_t}{M_t}\right)^{\frac{1}{\chi}}, \tag{17}$$

$$p_t^S = \frac{\partial Y_t}{\partial S_t} = (1 - \gamma) \left(\frac{Y_t}{S_t} \right)^{\frac{1}{\chi}}. \tag{18}$$

2.3.2 Manufacturing intermediate good

The manufacturing good is produced by combining capital k_t with employment n_t^M,

$$M_t = \left[\zeta k_t^{\frac{\alpha-1}{\alpha}} + (1 - \zeta)(n_t^M)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}}, \tag{19}$$

where ζ denotes the weight attached to capital versus labor and α is the elasticity of substitution.

An increase in ζ makes output more capital-intensive at the expense of labor, representing in our setup an increased robot adoption (automation). The microeconomic foundations are derived by Acemoglu and Restrepo (2018) in a framework where a continuum of tasks is used in production. Automation in that context is interpreted as a shift in the share of tasks that can be produced with capital. Acemoglu and Restrepo (2018) show how one can aggregate the tasks to establish a production function with aggregate capital and labor inputs (see also the discussion in Bergholt et al. (2020)).

Firms maximize the discounted expected value of future profits subject to the technology and the law of motion of employment (2). That is, they take the number of workers currently employed n_t^j as given and choose the number of vacancies to post v_t^j so as to employ the desired number of workers next period n_{t+1}^j. The firm also chooses the amount of capital to demand. The manufacturing firm solves the problem,

$$Q^M(n_t^M) = \max_{v_t^M, k_t} \left\{ p_t^M M_t - w_t^M n_t^M - r_t k_t - \kappa^M v_t^M + E_t \left[\Lambda_{t,t+1} Q^M(n_{t+1}^M) \right] \right\}, \tag{20}$$

where κ^M denotes the marginal cost of posting a vacancy. As the household owns the firm, the term $\Lambda_{t,t+1} = \beta \lambda_{ct+1} / \lambda_{ct}$ refers to the household's stochastic discount factor in which λ_{ct} denotes the Lagrange multiplier for the household budget constraint and β is the household's discount factor.

The first-order conditions with respect to v_t^M and k_t are,

$$\kappa^M = \psi_t^{fM} \times E_t \Lambda_{t,t+1} \left[p_{t+1}^M (1 - \zeta) \left(\frac{M_{t+1}}{n_{t+1}^M} \right)^{\frac{1}{\alpha}} - w_{t+1}^M + \frac{(1 - \sigma^M) \kappa^M}{\psi_{t+1}^{fM}} \right], \tag{21}$$

$$r_t = p_t^M \cdot \zeta \left(\frac{M_t}{k_t} \right)^{\frac{1}{\alpha}}. \tag{22}$$

Equation (21) states that the marginal cost of hiring a worker should equal the expected marginal benefit subject to the vacancy-filling probability. The latter includes the net value of the marginal product of labor, where ζ enters with a negative sign, minus the wage plus the continuation value. Equation (22) states that the return on capital is equal to the value of its marginal product, where ζ enters with a positive sign.

The value of the marginal job for the firm is given by,

$$V_{n^M t}^f = p_t^M (1 - \zeta) \left(\frac{M_t}{n_t^M} \right)^{\frac{1}{\alpha}} - w_t^M + \frac{\left(1 - \sigma^M\right) \kappa^M}{\psi_t^{fM}}. \tag{23}$$

2.3.3 Service intermediate good

In the service sector, we assume a simple production function with labor only,

$$S_t = B(n_t^S)^b, \tag{24}$$

where B denotes total factor productivity (TFP) and b is the degree of returns to scale.

A firm operating in this sector solves the following problem,

$$Q_t^S(n_t^S) = \max_{v_t^S} \left\{ p_t^S S_t - w_t^S n_t^S - \kappa^S v_t^S + E_t \left[\Lambda_{t,t+1} Q_{t+1}^S(n_{t+1}^S) \right] \right\}. \tag{25}$$

The first-order condition is,

$$\kappa^S = \psi_t^{fS} E_t \Lambda_{t,t+1} \left[p_{t+1}^S b \frac{S_{t+1}}{n_{t+1}^S} - w_{t+1}^S + \frac{\left(1 - \sigma^S\right) \kappa^S}{\psi_{t+1}^{fS}} \right]. \tag{26}$$

The value to the firm of a marginal job is given by,

$$V_{n^S t}^f = p_t^S b \frac{S_t}{n_t^S} - w_t^S + \frac{\left(1 - \sigma^S\right) \kappa^S}{\psi_t^{fS}}. \tag{27}$$

2.4 Wage bargaining

Following standard practice, the Nash bargaining problem in each sector is to maximize the weighted sum of log surpluses,

$$\max_{w_t^j} \left\{ \left(1 - \vartheta^j\right) \ln V_{n^j t}^h + \vartheta^j \ln V_{n^j t}^f \right\},$$

where ϑ^j denotes the bargaining power of firms and $V_{n^j t}^h$, $V_{n^j t}^f$ have been defined above. The first-order condition with respect to w_t^j is

$$\vartheta^j V_{n^j t}^h = \left(1 - \vartheta^j\right) \lambda_{ct} V_{n^j t}^f.$$

Through the derivations shown in the Online Appendix, we obtain the equilibrium values for wages in the two sectors,

$$w_t^M = \left(1 - \vartheta^M\right) \left(p_t^M (1 - \zeta) \left(\frac{M_t}{n_t^M}\right)^{\frac{1}{\alpha}} + \frac{\left(1 - \sigma^M\right) \kappa^M}{\psi_t^{fM}} \right) + \frac{\vartheta^M}{\lambda_{ct}} (\Phi l_t^{-\varphi} - \left(1 - \sigma^M\right) \lambda_{n^{M_t}}), \tag{28}$$

$$w_t^S = \left(1 - \vartheta^S\right) \left(p_t^S b \frac{S_t}{n_t^S} + \frac{\left(1 - \sigma^S\right) \kappa^S}{\psi_t^{fS}} \right) + \frac{\vartheta^S}{\lambda_{ct}} (\Phi l_t^{-\varphi} - \left(1 - \sigma^S\right) \lambda_{n^{S_t}}). \tag{29}$$

2.5 Resource constraint

The final good is used for consumption and investment, and also to cover vacancy costs.

$$Y_t = c_t + i_t + \kappa^M v_t^M + \kappa^S v_t^S. \tag{30}$$

The derivation of the resource constraint is shown in the Online Appendix.

3 Relative Labor Demand and Labor Supply in Equilibrium

In this section, we establish the equilibrium relationship between relative labor demand and relative labor supply in the two sectors.

Proposition 1. *In equilibrium, the sectoral ratio of labor market tightness depends only on the bargaining power and vacancy costs in the two sectors,*

$$\frac{\theta_t^M}{\theta_t^S} = \frac{\vartheta^M}{\left(1 - \vartheta^M\right)} \frac{\left(1 - \vartheta^S\right)}{\vartheta^S} \cdot \frac{\kappa^S}{\kappa^M}$$

Proof. See the Appendix. $\qquad\qquad\square$

Proposition 1 establishes that the *relative* labor market tightness of the two sectors is constant in equilibrium and characterizes its level. Asymmetric bargaining

power and/or vacancy costs introduce a wedge in tightness between the two sectors. Conversely, if both the bargaining power and vacancy costs are symmetric, tightness is equal in the two sectors. The derivation of Proposition 1 (see the Appendix) builds on Ravn (2008), where a relationship between tightness and the marginal utility of consumption is derived in a one-sector search and matching model with endogenous participation.

The relationship between relative labor supply and relative labor demand directly follows from the proposition,

$$
\underbrace{\frac{s}{1-s} \equiv \frac{u^M}{u^S}}_{\substack{\text{Relative labor} \\ \text{supply}}} = \frac{(1-\vartheta^M)}{\vartheta^M} \frac{\vartheta^S}{(1-\vartheta^S)} \frac{\kappa^M}{\kappa^S} \cdot \underbrace{\frac{v^M}{v^S}}_{\substack{\text{Relative labor} \\ \text{demand}}}
$$

For a given level of relative labor demand (which depends, among others, on the degree of automation ζ), the pool of job seekers in manufacturing increases with the relative (i) bargaining power of workers and (ii) vacancy cost. In the second case, an increased pool of unemployed is required to compensate for the higher vacancy cost when firms decide about new vacancies so that the level of labor demand is sustained in equilibrium.

Finally, notice that the household decides how to allocate job seekers by comparing the discounted expected values of searching in the two sectors, $\psi^{j,h}\beta E_t\left[V^h_{n^{j_{t+1}}}\right]$, which, in turn, is equal to the probability of finding a job times the discounted expected value of being employed. The optimal value s^* is given by,

$$
s^* = \begin{cases} 1 & \psi^{M,h}_t\beta E_t\left[V^h_{n^{M_{t+1}}}\right] > \psi^{S,h}_t\beta E_t\left[V^h_{n^{S_{t+1}}}\right] \\ s^* \in (0,1) & \psi^{M,h}_t\beta E_t\left[V^h_{n^{M_{t+1}}}\right] = \psi^{S,h}_t\beta E_t\left[V^h_{n^{S_{t+1}}}\right] \\ 0 & \psi^{M,h}_t\beta E_t\left[V^h_{n^{M_{t+1}}}\right] < \psi^{S,h}_t\beta E_t\left[V^h_{n^{S_{t+1}}}\right] \end{cases}
$$

In general equilibrium, we can rule out the two corner solutions. If $s^* = 1$ and all the unemployed search in manufacturing, there is no production in services. Yet, as long as the two sectoral goods are not perfect substitutes in the final good production, the marginal product of the service good becomes infinite, leading to an infinite wage, which is incompatible with zero labor supply in this sector. If $s^* = 0$ and all the unemployed search in services, there is no production in manufacturing. Yet, as long as capital and labor are not perfect substitutes in manufacturing production, the marginal product of labor in manufacturing becomes infinite, which, again, is incompatible with a zero supply of labor in that sector. Therefore, the only possible solution is $s^* \in (0,1)$.

4 Calibration Strategy

In this section, we describe the calibration of the initial steady state, which we take to refer to the start year 1994 in the analysis of Dauth et al. (2021). We calibrate the model annually to the German economy. Table I summarizes our calibration.

Household. We use the data set built by Jordà et al. (2019) to compute the return to capital r in Germany, equal to 5% in 1994. We set the capital depreciation rate δ equal to 4%. To choose the value for the discount factor, we use the Euler equation in the steady state, $\beta = 1/(1 + r - \delta)$. For the inverse elasticity of the intertemporal substitution η, much of the literature uses econometric estimates between 0 and 2 (see, e.g., Hansen and Singleton (1983)). The estimated aggregate Frisch elasticity for Germany varies between 0.85 and 1.06 in a micro panel of men in Germany from 2000 to 2013 used by Kneip et al. (2020). We thus set the Frisch elasticity to 0.85 ($\phi = 2$). We have performed sensitivity analysis for different values $\phi = 4, 6$ (see the Online Appendix and footnote 13). We calibrate the relative utility weight for leisure Φ to target a participation rate of 70%.

Production. To calibrate the parameters of the aggregate production function, we follow Iftikhar and Zaharieva (2019), setting the share of manufacturing output γ to 0.33 and the elasticity of substitution between the manufacturing and the service goods χ to 0.3. In the manufacturing production function, we set the elasticity of substitution between capital and labor α to 0.6. Based on a meta-regression sample, Knoblach et al. (2020) estimate a long-run elasticity for the aggregate economy in the range of 0.45-0.87, noting that most industrial estimates do not deviate significantly from the estimate for the aggregate economy. Our calibrated value is also in line with Oberfield and Raval (2020) who find the US manufacturing sector's aggregate elasticity to be in the range of 0.5-0.7. Most of the literature estimates constant (or slightly decreasing) returns at the industry level (see, e.g., Ahmad et al. (2019) and Maioli (2004)). Therefore, we set the parameter b, in the production function of the service good, equal to one. We also normalize the TFP parameter B to one.

Labor Markets. To calibrate the parameters for the bargaining power of firms in each sector, we take weighted averages of the estimates for high-skill and low-skill workers in Iftikhar and Zaharieva (2019). A lower bargaining power for workers in the service sector is in line with the empirical evidence that service workers get a lower fraction of output produced in their sector, leading to a mild wage premium in manufacturing of around 2% in our calibration. The same authors estimate the

DESCRIPTION		VALUE	TARGET/SOURCE
HOUSEHOLD			
β	Discount factor	0.99	Return to capital, 5%
δ	Depreciation rate	0.04	Standard calibration
Φ	Relative utility from leisure	0.8	Participation Rate, 70%
ϕ	Inverse Frisch elasticity of labor supply	2	Kneip et al. (2020)
η	Inverse elasticity of intertemporal substitution	2	Hansen and Singleton (1983)
PRODUCTION			
γ	Share of manufacturing in total output	0.33	Iftikhar and Zaharieva (2019)
χ	Manufacturing-services elasticity of substitution	0.3	Iftikhar and Zaharieva (2019)
α	Capital-labor elasticity of substitution	0.8	Knoblach et al. (2020)
B	TFP in services	1	normalization
b	Degree of returns to scale in services	1	Ahmad et al. (2019)
LABOR MARKET			
θ^M, θ^S	Bargaining power of firms	0.43, 0.6	Iftikhar and Zaharieva (2019)
μ_1	Matching efficiency	0.58	Iftikhar and Zaharieva (2019)
μ_2	Elasticity of matching to vacancies	0.46	Iftikhar and Zaharieva (2019)
σ	Separation rate	0.08	Iftikhar and Zaharieva (2019)
κ	Vacancy cost	0.1	Share of the average wage, 20%
ϖ	Replacement rate	0.6	OECD data

Table I: Calibration

average job duration rate in Germany to be 12.25 years, so we set the destruction rate in both sectors as $\sigma = 1/12.25 = 0.08$. We set the gross replacement rate ϖ equal to 0.6.[4] For the vacancy cost parameter, we set in both sectors $\kappa = 0.1$, which implies that vacancy costs represent around 20% of the average wage. Using aggregate data of the Federal Employment Agency, Iftikhar and Zaharieva (2019) estimate the elasticity of the matching function with respect to vacancies to be 0.54, which is close to 0.5, often assumed in the search and matching literature. Their estimate for the matching efficiency parameter is 0.58.

5 Automation and Sectoral Reallocation

In this section, we present the main results of our quantitative analysis. First, we discuss steady-state comparative statics with respect to an increase in the degree of automation ζ. Then, we show that the model can replicate the magnitude of the decline in the ratio of manufacturing employment to service employment in Germany between 1994 and 2014.

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[4]According to the OECD, the standard rates in Germany after 2000 are 60% of the previous earnings net of tax.
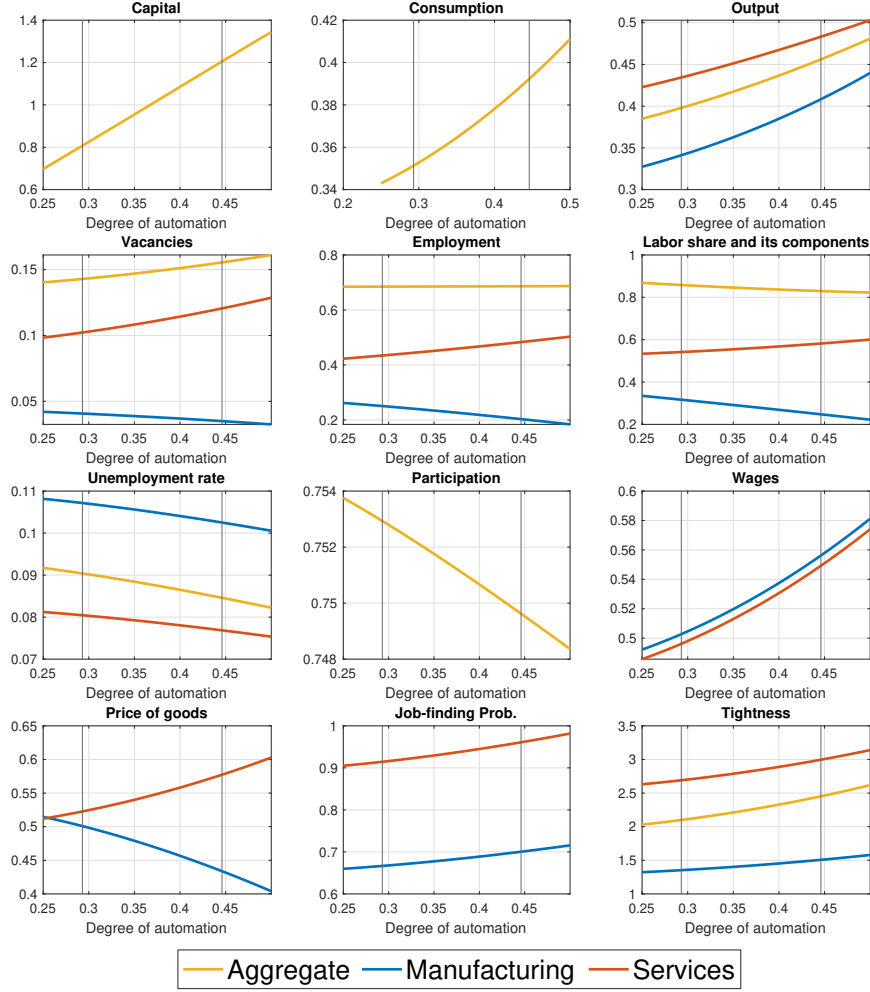
Figure 4: Steady-state effects of automation in a two-sector economy

Note: The y-axis shows steady-state levels. In each plot, the two vertical lines refer to the two steady states that we compare in Table II. In the labor share plot, the blue and red lines refer to the share in total output of employment compensation in manufacturing and services, respectively.

5.1 Analysis Across Steady States

Figure 4 depicts results for the steady-state levels of the main variables of the model for $0.25 < \zeta < 0.5$, which is an empirically relevant interval.

Sectoral Reallocation of Output. A higher degree of automation ζ corresponds in our model to an increased (decreased) capital (labor) intensity of manufacturing production. Since the steady-state return to capital is constant, while the steady-state return to labor can freely adjust, the capital increase due to a higher ζ dominates the labor decline. Therefore, manufacturing output increases.[5] In turn, output in services also increases since the two sectoral goods are complements in the pro-

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[5]The effect of an increase in ζ on manufacturing output M is expressed by the derivative:

duction of the final good *(sectoral complementarity effect)*. In addition, as the total output increases, the household who is the owner of capital and firms enjoys a higher income and demands more of the service good *(income effect)*. Therefore, the economy experiences an aggregate output expansion. Overall, a higher ζ increases the steady-state ratio of manufacturing to service output M/S and decreases the relative price of the manufacturing good (see equations (17) and (18)).[6]

Consumption, Participation, and Labor Share. The positive income effect for the household explains the increase in consumption and the decrease of participation. Automation has a negative effect on the aggregate labor share, which is driven by the manufacturing sector and is in line with the literature findings on the importance of the automation mechanism for a countercyclical labor share (see, e.g., Bergholt et al. (2020) and Leduc and Liu (2020b)).

Sectoral Reallocation of Labor. Vacancies in the manufacturing sector decrease. Automation affects labor demand in manufacturing through two competing channels: (a) production becomes less labor-intensive, which tends to decrease employment *(labor-intensity channel)* and (b) since capital and labor are complements, the increase in capital tends to increase labor demand *(capital-labor complementarity effect)*. Vacancies in services increase due to the *sectoral complementarity effect* and the positive *income effect*. Total vacancies increase as well.

The number of unemployed searchers drops in the manufacturing sector as households reduce participation and reallocate job search towards services. The unemployment rate drops in the service sector too, but the share of searchers increases (see blue line in Figure 5). Total unemployment falls.

Labor market tightness increases in both sectors. The effect on the hiring rates follows from the fact that they are a positive function of tightness (while the opposite holds for vacancy-filling rates). The impact of automation on wages in both sectors is positive, consistently with the decrease in the vacancy-filling probabilities.

Following the sectoral reallocation of labor, employment increases in services and falls in manufacturing in such a way that aggregate employment remains relatively

$$\frac{\partial M}{\partial \zeta} = \frac{1}{\alpha} M^{(1-\alpha)} \left[k^\alpha - (n^M)^\alpha + \zeta \alpha \frac{\partial k}{\partial \zeta} + (1-\zeta)\alpha \frac{\partial n^M}{\partial \zeta} \right]$$

An increase in ζ induces an accumulation of capital ($\frac{\partial k}{\partial \zeta} > 0$) and a decrease in employment ($\frac{\partial n^M}{\partial \zeta} > 0$). The difference $k^\alpha - (n^M)^\alpha$ also matters for which effect dominates. If the initial value of ζ is sufficiently low, the steady-state capital stock k is relatively low and labor n^M is relatively more important in the production, leading to a decrease in manufacturing output.

[6]Recall that capital serves as input only in manufacturing production.
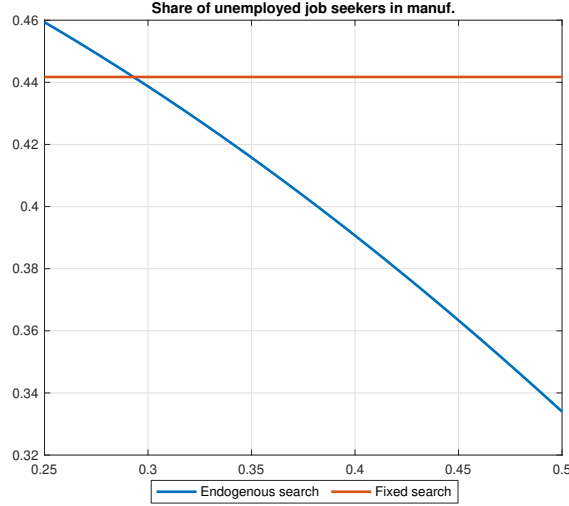
Share of unemployed job seekers in manuf.

Figure 5: The steady-state effect of automation on searchers' share in manufacturing

Note: The y-axis shows steady-state levels. The blue line refers to the baseline model, whereas the red line refers to a model variant where the sectoral allocation of job seekers is kept fixed.

constant, in line with the empirical evidence in Dauth et al. (2021). The pattern matches well the one observed in Figure 1.[7]

In sum, labor markets adjust to automation through vacancy creation, sectoral reallocation of the unemployed, and participation. The findings also highlight the expansionary effects of automation in the economy, namely the aggregate output expansion and unemployment reduction.

5.2 The Decline of the Sectoral Labor Ratio from 1994 to 2014

The model can also replicate the magnitude of the decline in the ratio of manufacturing employment to service employment in Germany. Specifically, we take from the data the values of the capital share in manufacturing in 1994 and 2014, which are the start and end years in the empirical analysis in Dauth et al. (2021).[8] Following Iftikhar and Zaharieva (2019), we define our manufacturing sector as the aggregate of Industries A-F in the German WZ08 industry classification. Moreover, robots are predominantly employed in these industries. Then, we compute the values of the degree of automation ζ that generate these two values in our model. For a manufacturing capital share equal to 0.24 in 1994, we find that the implied value of ζ is 0.29, while for a capital share equal to 0.36 in 2014 the implied value of ζ is 0.44 (see Table II).

Next, we examine the steady-state values for the ratio of manufacturing em-

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[7]To also match the levels, we would need to add capital in the service sector.

[8]EUKLEMS defines the capital share as the ratio of capital services to the value added.

ployment to service employment for these two values of ζ. The model predicts a decline of 27% in the ratio of manufacturing employment to service employment, which is reasonably close to the one found in the aggregate data for the German economy (32%). Using a local labor market approach, Dauth et al. (2021) find that, on average, employment in manufacturing falls by 16.86%, while non-manufacturing employment increases by 3.74%.[9] This implies that the weighted average of the sectoral labor ratio over the 402 local labor markets analyzed in their paper decreases by 19.85%.[10] Therefore, our model's prediction about a decline of 27% lies between the value estimated using our aggregated data (32%) and the statistics for local labor markets (nearly 20%) in Dauth et al. (2021).

Variable	Notation	1994	2014	Change: model	Change: data
Degree of automation	ζ	0.293	0.446	52%	N/A
Manufacturing capital share	$\frac{rK}{p^M M}$	0.236	0.340	44%	44%
Labor ratio: manuf./service	$\frac{n^M}{n^S}$	0.576	0.420	-27%	-32%

Table II: Comparison of two steady states (Germany 1994 and 2014)

6 What Determines the Extent of Sectoral Reallocation?

In this section, we investigate the role of key parameters and features of the model, namely (i) the elasticity of substitution between the sectoral goods, (ii) the elasticity of substitution between capital and labor, and (iii) the sectoral mobility of job seekers.

6.1 Elasticities of Substitution

Between the Sectoral Goods. The elasticity of substitution between the sectoral goods χ matters both for the sectoral reallocation of output and for the sectoral reallocation of labor. Figure 6 compares the change in key sectoral ratios of variables as the degree of automation ζ increases from an initial steady state (with $\zeta = 0.25$) for a higher elasticity χ and for our benchmark calibration. Additional variables and the same results in levels of these ratios are included in the Online Appendix. Relative to the baseline calibration ($\chi = 0.3$), when we increase the

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[9]See Table 1 in Dauth et al. (2021).

[10]We computed the rate of change in $\frac{n^M}{n^S}$ as: $\widehat{\frac{n^M}{n^S}} = \frac{1+\widehat{n^M}}{1+\widehat{n^S}} - 1$, where $\widehat{x} = \frac{x_{2014}-x_{1994}}{x_{1994}}$.

elasticity ($\chi = 1.5$), the sectoral output ratio M/S changes by more due to automation because it is easier now to substitute services by manufacturing intermediate goods in the final good production.[11] Consequently, an increase in χ mitigates the effect of automation on the sectoral reallocation of labor, vacancies, and job seekers (see the plots of the sectoral labor ratios n^M/n^S, v^M/v^S, and u^M/u^S). In line with these results, the drop in the wage premium in manufacturing w^M/w^S becomes less pronounced.

Between Capital and Labor. The elasticity of substitution between capital and labor matters for the sectoral reallocation of labor. Figure 6 also depicts results for a lower elasticitity of substitution between capital and labor α. Through the capital-labor complementarity channel, a decrease in α tends to dampen the automation-driven sectoral reallocation of vacancies, job seekers, and labor as well as the drop in the wage premium in manufacturing (see the plots of the sectoral labor ratios v^M/v^S, u^M/u^S, n^M/n^S, and w^M/w^S). It also affects the sectoral price ratio (p^M/p^S) reaction to automation.

6.2 Sectoral Mobility of Job Seekers

Next, we explore the extent to which shutting down the reallocation of job seekers between the two sectors affects our findings. We examine the comparative statics with (a) endogenous sector-specific search and (b) fixed sectoral shares of job seekers by keeping the share of searchers in manufacturing s equal to the value it attains endogenously in the initial calibrated steady state of Section 5.2 $\zeta = 0.293$ (see Figure 5). In other words, equation (13) is no longer used. Hence, although the number of employees per sector can evolve separately through the dynamics of vacancy postings, matches, and participation, households cannot freely reallocate job seekers between sectors.

With a fixed sectoral allocation of job seekers, as we move from a steady state with $\zeta = 0.293$ to a steady state with $\zeta = 0.446$ (in line with Table II), total employment changes even less than with endogenous allocation (see Figure 7).[12] If job seekers cannot move, the unemployment rate in manufacturing increases with ζ. At the same time, the negative effect on the unemployment rate in services becomes sharper since without the reallocation of job seekers there is less competition in this labor market. Yet, differences are not very large in magnitude.

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[11] As shown in the Online Appendix, even when the two goods are imperfect substitutes ($\chi = 1.5$), output in services increases due to the income effect.

[12] Figure 7 omits the output and labor share variables as the differences between the two model variants are minimal. Results are available upon request.
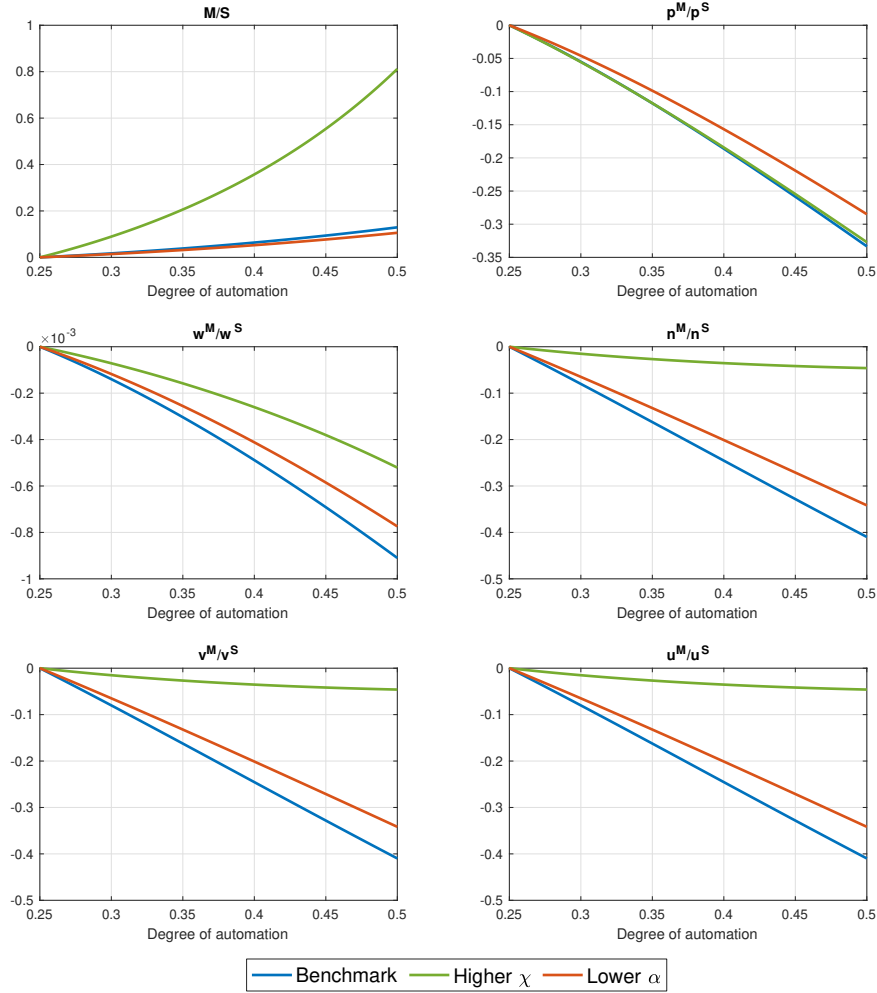
Figure 6: Steady-state effects of automation in a two-sector economy: Different elasticities of substitution between capital and labor ($\alpha = 0.7$) and between the two goods ($\chi = 1.5$)

Note: All the plotted variables are normalized to 0 in the steady state with $\zeta = 0.25$. We denote the ratios of manufacturing to services variables as follows: M/S for output, p^M/p^S for prices, w^M/w^S for wages, n^M/n^S for labor, v^M/v^S for vacancies, and u^M/u^S for job seekers.

The sectoral mobility of job seekers also matters for the effect of automation on vacancies: under fixed search, the impact on manufacturing vacancies becomes more negative, while the positive effect on vacancies in services is reinforced. This result is explained by the effects on sectoral prices, which, in turn, suggest that the sectoral reallocation of output is somewhat smaller than in the baseline model.[13]

7 Conclusion

The paper studies the sectoral impact of automation through the lens of a general equilibrium model with matching frictions, endogenous participation, and two production sectors. In the model, as in empirical evidence from Germany (see Dauth et al. (2021)), automation induces firms to create fewer new vacancies and job seekers to search less in the robot-exposed sector. Analysis across steady states shows that the reduction in manufacturing employment from automation can be offset by the increased service employment, thus leaving aggregate employment unaffected. The model does a good job in replicating (a) qualitatively the empirical evolution of employment and employees' compensation (as a share of GDP) in manufacturing and services, and (b) the magnitude of the decline in the ratio of manufacturing employment to service employment from 1994 to 2014. Our findings also highlight the expansionary impact of automation on aggregate output.

Our model can be extended along several dimensions. For instance, the good produced in the automated sector (manufacturing) is, in fact, a tradable good. One plausible extension could therefore be to consider the sectoral impact of automation in an open economy framework. Another interesting avenue for further research would be to introduce skill heterogeneity and capital-skill complementarity (see, e.g., Dolado et al. (2020b), Santini (2021)). Such a setup could capture the idea that robots are complements with high-skill workers but substitutes for low-skill workers, allowing to study implications for inequality. We leave these topics for future research.

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[13]In the Online Appendix, we also show results for different values of the parameter governing the Frisch elasticity of labor supply ($\phi = 4, 6$). A lower value of the Frisch elasticity (higher value of ϕ) matters for the steady-state levels of the variables but without affecting our main results.
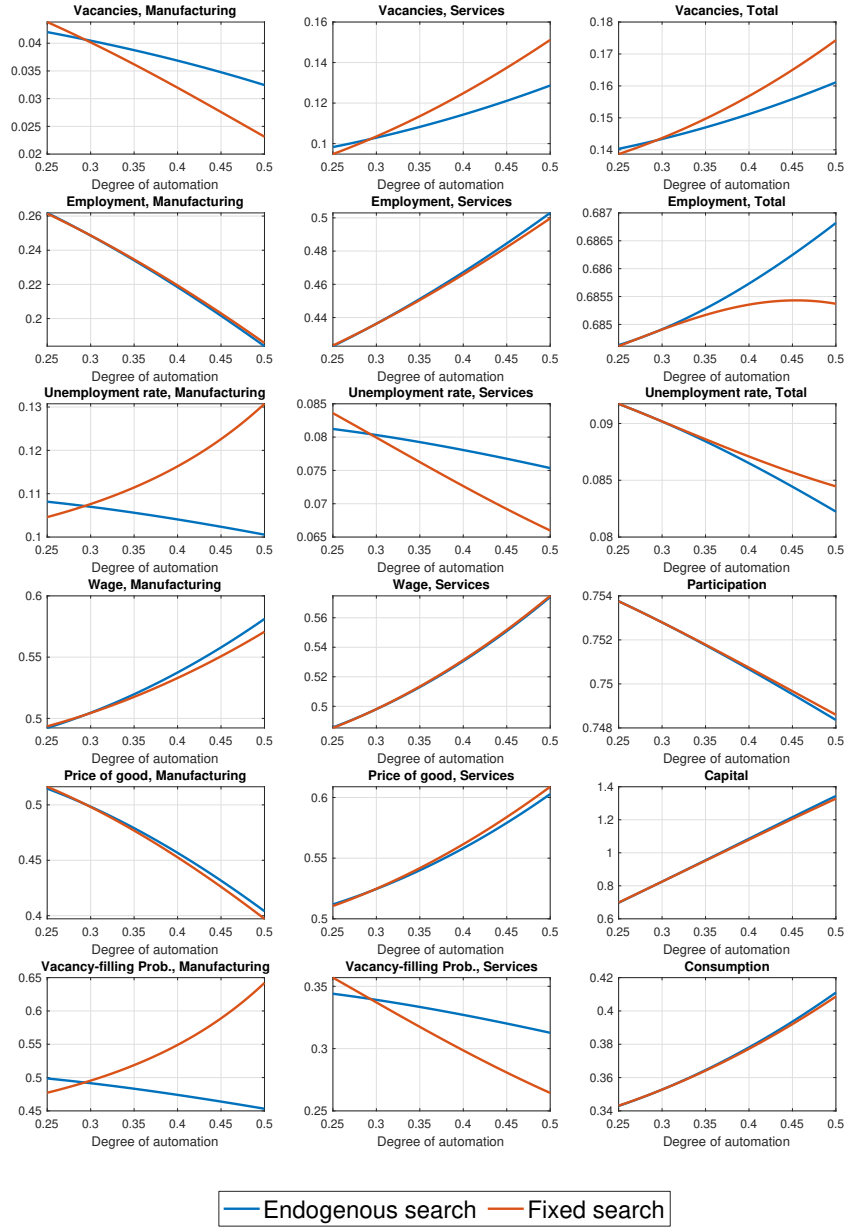
Figure 7: Steady-state effects of automation with and without sectoral mobility

Note: The y-axis shows steady-state levels. The blue line refers to the baseline model, whereas the red line refers to a model variant where the sectoral allocation of job seekers is kept fixed.

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Appendix: Proof of Proposition 1

Proof. From the maximization problem of the household we have,

$$\Phi l_t^{-\varphi} = \lambda_{n^{M_t}} \psi_t^{hM} s_t + \lambda_{n^{S_t}} \psi_t^{hS} (1 - s_t) + \lambda_{ct} \bar{b}_t, \tag{A.1}$$

and

$$\lambda_{n^{M_t}} \psi_t^{hM} = \lambda_{n^{S_t}} \psi_t^{hS}. \tag{A.2}$$

We can substitute (A.2) into (A.1) and obtain,

$$\Phi l_t^{-\varphi} = \lambda_{n^{S_t}} \psi_t^{hM} + \lambda_{ct} \bar{b}_t,$$

or alternatively we can get,

$$\Phi l_t^{-\varphi} = \lambda_{n^{S_t}} \psi_t^{hS} + \lambda_{ct} \bar{b}_t,$$

which states that the marginal utility of leisure is equal to the value of being unemployed. The latter in turn is equal to the utility value of the unemployment benefit plus the probability of finding a job times the value of being employed. We invert these equations and obtain,

$$\lambda_{n^{M_t}} = \frac{\Phi l_t^{-\varphi} - \lambda_{ct} \bar{b}_t}{\psi_t^{hM}},$$

and

$$\lambda_{n^{S_t}} = \frac{\Phi l_t^{-\varphi} - \lambda_{ct} \bar{b}_t}{\psi_t^{hS}}.$$

The values of an additional unit of employment in the two sectors are,

$$V_{n^{M_t}}^h = \lambda_{ct} w_t^M - \Phi l_t^{-\varphi} + \left(1 - \sigma^M\right) \lambda_{n^{M_t}},$$

and

$$V_{n^{S_t}}^h = \lambda_{ct} w_t^S - \Phi l_t^{-\varphi} + \left(1 - \sigma^S\right) \lambda_{n^{S_t}}.$$

The Lagrange multipliers λ_{n^M} and λ_{n^S} are equal to,

$$\lambda_{n^{M_t}} = \beta E_t \left[\lambda_{c,t+1} w_{t+1}^M - \Phi l_{t+1}^{-\varphi} + \lambda_{n^{M_{t+1}}} (1 - \sigma^M) \right],$$

and

$$\lambda_{n^{S_t}} = \beta E_t \left[\lambda_{c,t+1} w_{t+1}^S - \Phi l_{t+1}^{-\varphi} + \lambda_{n^{S_{t+1}}} (1 - \sigma^S) \right].$$

Therefore, we can write,

$$\lambda_{n^S t} = \beta E_t \left[V^h_{n^S t+1} \right], \tag{A.3}$$

and

$$\lambda_{n^M t} = \beta E_t \left[V^h_{n^M t+1} \right]. \tag{A.4}$$

Consider now the problems of the two representative firms where the first-order conditions with respect to vacancies are given by,

$$\frac{\kappa^M}{\psi_t^{fM}} = E_t \Lambda_{t,t+1} \left[p^M_{t+1}(1-\zeta) \left(\frac{M_{t+1}}{n^M_{t+1}} \right)^{\frac{1}{\alpha}} - w^M_{t+1} + \frac{(1-\sigma^M)\kappa^M}{\psi_{t+1}^{fM}} \right],$$

and

$$\frac{\kappa^S}{\psi_t^{fS}} = E_t \Lambda_{t,t+1} \left[p^S_{t+1} b \frac{S_{t+1}}{n^S_{t+1}} - w^S_{t+1} + \frac{(1-\sigma^S)\kappa^S}{\psi_{t+1}^{fS}} \right].$$

The marginal value of an extra unit of employment in period t for each sector is,

$$V^f_{n^M t} = p^M_t (1-\zeta) \left(\frac{M_t}{n^M_t} \right)^{\frac{1}{\alpha}} - w^M_t + \frac{(1-\sigma^M)\kappa^M}{\psi_t^{fM}},$$

and

$$V^f_{n^S t} = p^S_t b \frac{S_t}{n^S_t} - w^S_t + \frac{(1-\sigma^S)\kappa^S}{\psi_t^{fS}}.$$

Therefore, we can write,

$$\frac{\kappa^M}{\psi_t^{fM}} = E_t \Lambda_{t,t+1} \left[V^f_{n^M t+1} \right], \tag{A.5}$$

and

$$\frac{\kappa^S}{\psi_t^{fS}} = E_t \Lambda_{t,t+1} \left[V^f_{n^S t+1} \right].$$

Recall that the first-order conditions of the wage bargaining problems are,

$$\vartheta^M V^h_{n^M t} = \left(1 - \vartheta^M \right) \lambda_{ct} V^f_{n^M t}, \tag{A.6}$$

and

$$\vartheta^S V^h_{n^S t} = \left(1 - \vartheta^S \right) \lambda_{ct} V^f_{n^S t}.$$

By evaluating equation (A.6) for the next period, multiplying by $\frac{\beta}{\lambda_{c,t}}$, and taking expectations we obtain,

$$\frac{\vartheta^M}{\lambda_{c,t}} \beta E_t \left[V^h_{n^M t+1} \right] = \left(1 - \vartheta^M \right) E_t \Lambda_{t,t+1} \left[V^f_{n^M t+1} \right].$$

Substituting (A.4) and (A.5) we get,

$$\frac{\vartheta^M}{\lambda_{c,t}} \frac{\left(\Phi l_t^{-\varphi} - \lambda_{ct}\bar{b}_t\right)}{\psi_t^{hM}} = \left(1 - \vartheta^M\right) \frac{\kappa^M}{\psi_t^{fM}},$$

and, after rearranging terms, we obtain,

$$\theta_t^M = \frac{\vartheta^M}{1 - \vartheta^M} \frac{\left(\Phi l_t^{-\varphi} - \lambda_{ct}\bar{b}_t\right)}{\kappa^M}.$$

Similarly for the service sector, we have,

$$\theta_t^S = \frac{\vartheta^S}{1 - \vartheta^S} \frac{\left(\Phi l_t^{-\varphi} - \lambda_{ct}\bar{b}_t\right)}{\kappa^S}.$$

These relations are similar to the the linear relationship between labor market tightness and the marginal utility of consumption derived by Ravn (2008) in a one-sector search and matching model with endogenous participation. By taking the ratio of tightness in the two sectors, we obtain the relationship of Proposition 1.

$$\frac{\theta_t^M}{\theta_t^S} = \frac{\frac{\vartheta^M}{1-\vartheta^M}}{\frac{\vartheta_S}{1-\vartheta_S}} \cdot \frac{\kappa^S}{\kappa^M}.$$

$\square$