Emigration and Fiscal Austerity in a Depression

Online Appendix

*(not intended for publication)*

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1 Graphical Illustration of the Model

Figure 1: Overview of the Model

(a) Household members: Residents and Migrants

(b) Firms

S&M denotes search and matching; K,L denote capita and labour, respectively.
2 Model Equations

2.1 The Household’s Problem

The household’s Lagrangean can be written as

\[ L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t - \zeta \tilde{C}_t)^{1-\eta}}{1-\eta} - \chi \frac{h_t^{1+\xi} + h_e^{1+\xi} n_{e,t}}{1+\xi} - \Omega \frac{(n_{e,t})^{1+\mu}}{1+\mu} \right\} - \lambda_{c,t} \left( (1 + \tau^c) c_t + \frac{b_{g,t+1}}{r_t} - \frac{e_t b_{f,t+1}}{r_{f,t}} + \phi(z_t) n_t + \zeta (\tilde{s}_t \tilde{u}_t) s_t u_t - (1 - \tau^n) w_t h_t n_t - b u_t \right) \]

\[ - \lambda_{k,t} \left[ (1 - \tau^k) k_{t+1} - \frac{b_{g,t+1}}{r_t} + \frac{e_t b_{f,t+1}}{r_{f,t}} - \Pi_t - T \right] \]

\[ - \lambda_{n,t} \left[ n_{t+1} - (1 - \sigma - \psi_H \varphi(z_t)) n_t - \psi_H (1 - s_t) u_t \right] \]

\[ - \lambda_{e,t} \left[ e_t (1 + \tau^e) e_{c,t} - (1 - \tau^n) w^* n_{e,t} \right] \]
\[
\begin{align*}
\lambda_{k,t} \tilde{d} (x_t)^{t-1} & = \lambda_{c,t} \left\{ r_t^k - \tau^k (r_t^k - (1 + \iota) \delta_t) \right\}, \quad (4) \\
b_{g,t+1} & : \\
1/\beta & = \frac{E_t \lambda_{c,t+1} r_t}{\lambda_{c,t}}. \quad (5) \\
b_{f,t+1} & : \\
1/\beta & = \frac{E_t \lambda_{c,t+1} e_{t+1}}{\lambda_{c,t} e_t} r_{f,t}. \quad (6) \\
n_{t+1} & : \\
\lambda_{n,t}/\beta & = E_t \lambda_{c,t+1} \left[ (1 - \tau^n) w_{t+1} h_{t+1} - \phi (z_{t+1}) \right] - E_t \lambda_{u,t+1} - \chi \frac{h_{t+1}^{1+\xi}}{1 + \xi} \\
& + E_t \lambda_{n,t+1} (1 - \sigma - \psi_H^* \varphi (z_{t+1})) + E_t \lambda_{e,t+1} \psi_H^* \varphi (z_{t+1}). \quad (7) \\
n_{e,t+1} & : \\
\lambda_{e,t}/\beta & = E_t \lambda_{c,t+1} \left( (1 - \tau^n*) e_{t+1} w^* h_e - \chi \frac{h_{t+1}^{1+\xi}}{1 + \xi} - \Omega (n_{e,t+1})^\mu - E_t \lambda_{u,t+1} \right) \\
& + E_t \lambda_{e,t+1} (1 - \sigma^*) . \quad (8) \\
u_t & : \\
\lambda_{u,t} & = \lambda_{c,t} (b - \varsigma (\tilde{s}_t \tilde{u}_t) s_t) + \lambda_{n,t} \psi_{H,t} (1 - s_t) + \lambda_{e,t} \psi_H^* s_t. \quad (9) \\
s_t & : \\
\lambda_{e,t} \psi_H^* - \lambda_{c,t} \varsigma (\tilde{s}_t \tilde{u}_t) & = \lambda_{n,t} \psi_{H,t}. \quad (10) \\
z_t & : \\
\lambda_{c,t} \frac{\phi'(z_t)}{\varphi'(z_t)} & = \psi_H^* (\lambda_{c,t} - \lambda_{n,t}). \quad (11) \\
\end{align*}
\]

Combining equations (9) and (10) we get the following two expressions
\[
\lambda_{u,t} = \lambda_{c,t} b + \lambda_{n,t} \psi_{H,t}, \quad (12)
\]
\begin{align*}
\lambda_{u,t} &= \lambda_{c,t} (b - \varsigma (\tilde{s}_t \tilde{u}_t)) + \lambda_{e,t} \psi_H^*. 
\end{align*}

(13)

We then use (12) to replace \( \lambda_{u,t+1} \) in (7),

\begin{align*}
\lambda_{n,t}/\beta &= E_t \lambda_{c,t+1} [(1 - \tau^n) w_{t+1} h_{t+1} - b - \phi (z_{t+1})] - \chi \frac{h_{t+1}^{1+\xi}}{1 + \xi} \\
&\quad + E_t \lambda_{n,t+1} (1 - \sigma - \psi_{H,t+1} - \psi_H^* \varphi (z_{t+1})) + E_t \lambda_{e,t+1} \psi_H^* \varphi (z_{t+1}) .
\end{align*}

(14)

and we use (13) to replace \( \lambda_{u,t+1} \) in (8),

\begin{align*}
\lambda_{e,t}/\beta &= E_t \lambda_{c,t+1} \left( (1 - \tau^n*) e_{t+1} w^* h_e - b + \varsigma (\tilde{s}_{t+1} \tilde{u}_{t+1}) \right) - \chi \frac{h_{t+1}^{1+\xi}}{1 + \xi} - \Omega (n_{e,t+1})^\mu \\
&\quad + E_t \lambda_{e,t+1} (1 - \sigma^* - \psi_H^*) .
\end{align*}

(15)

The last two expressions correspond to equations (14) and (15) in the paper.

2.2 The Wage-Hours Bargaining Problem

2.2.1 Household’s Surplus

The surplus for workers consists of the asset value of employment net of the outside option (value of being unemployed): \( S_H^t \equiv V_{t+1}^E - V_{t+1}^U \). The asset value of employment \( V_{t+1}^E \) is given by,

\begin{align*}
V_{t+1}^E &= (1 - \tau^n) w_t h_t - \phi (z_t) - \frac{\chi h_t^{1+\xi}}{\lambda_{c,t} 1 + \xi} \\
&\quad + E_t \beta_{t+1} \left\{ (1 - \sigma - \psi_H^* \varphi (z_t)) V_{t+1}^E + \sigma V_{t+1}^U + \psi_H^* \varphi (z_t) V_{t+1}^F \right\} .
\end{align*}

where the value of being unemployed at Home \( V_{t+1}^U \) is given by,

\begin{align*}
V_{t+1}^U &= b + E_t \beta_{t+1} \left\{ \psi_H V_{t+1}^E + (1 - \psi_{H,t}) V_{t+1}^U \right\} .
\end{align*}

Hence, the worker’s surplus \( S_H^t \) is given by,

\begin{align*}
S_H^t &= (1 - \tau^n) w_t h_t - \phi (z_t) - b - \frac{\chi h_t^{1+\xi}}{\lambda_{c,t} 1 + \xi} + E_t \beta_{t+1} (1 - \sigma - \psi_{H,t}) S_{t+1}^H \\
&\quad + E_t \beta_{t+1} \psi_H^* \varphi (z_t) \left\{ V_{t+1}^E - V_{t+1}^E \right\} .
\end{align*}

In the previous expression, \( V_{t+1}^E \) marks the value of being employed abroad,

\begin{align*}
V_{t+1}^E &= e_t (1 - \tau^n*) w^* h_e - \frac{\chi h_e^{1+\xi}}{\lambda_{c,t} 1 + \xi} - \frac{\Omega (n_{e,t})^\mu}{\lambda_{c,t}} \\
&\quad + E_t \beta_{t+1} \left\{ (1 - \sigma^*) V_{t+1}^E + \sigma^* V_{t+1}^U \right\} .
\end{align*}
where we assume that when an emigrant looses her job she joins the stock of job seekers searching for a job abroad. The value of job seeking abroad $V_{t}^{UF}$ is given by,

$$V_{t}^{UF} \equiv b - \varsigma (\bar{s}_t \bar{u}_t) + E_t \beta_{t+1} \{ \psi_H^* V_{t+1}^{EF} + (1 - \psi_H^*) V_{t+1}^{UF} \}.$$ 

Hence, migrant workers’ surplus $S_{h,t}^F \equiv V_{t}^{EF} - V_{t}^{UF}$ is given by,

$$S_{h,t}^F = e_t (1 - \tau^n) w^* h_e - b - \varsigma (\bar{s}_t \bar{u}_t) - \frac{\chi h_e^{1+\xi}}{\lambda_{c,t} 1 + \xi} - \frac{\Omega}{\lambda_{c,t}} (n_{e,t})^n + (1 - \sigma^* - \psi_F^*) E_t \beta_{t+1} S_{h,t+1}^F.$$

Optimality implies that the value of job seeking at home or abroad must be equal (see equation (9)). Hence, $V_{t}^{UH} = V_{t}^{UF}$ implies,

$$\psi_{H,t} E_t \beta_{t+1} S_{h,t+1}^H = \psi_{H}^* E_t \beta_{t+1} S_{h,t+1}^F - \varsigma (\bar{s}_t \bar{u}_t).$$

### 2.2.2 Firm’s Surplus

For the firm, the surplus from a match is given by,

$$S_{t}^F = (1 - \alpha) \frac{p_{yt} y_t}{n_t} - w_t h_t + (1 - \sigma - \psi_H^* \varphi (z_t)) E_t \beta_{t+1} S_{t+1}^F,$$

which, using equation (9) can be written as,

$$S_{t}^F = (1 - \alpha) \frac{p_{yt} y_t}{n_t} - w_t h_t + (1 - \sigma - \psi_H^* \varphi (z_t)) \frac{\kappa}{\psi_{F,t}}.$$

### 2.2.3 The Nash-Bargained Wage

Inserting the two surpluses into the splitting rule $(1 - \vartheta) (1 - \tau^n) S_{t}^F = \vartheta S_{t}^H$ and solving for the wage yields,

$$w_t h_t = (1 - \vartheta) \left\{ (1 - \alpha) \frac{p_{yt} y_t}{n_t} + (1 - \varphi (z_t)) \frac{\psi_{H,t}}{\psi_{F,t}} \kappa \right\} + \frac{\vartheta}{(1 - \tau^n)} \left\{ b + \frac{\chi h_t^{1+\xi}}{\lambda_{c,t} 1 + \xi} + \phi (z_t) - \varphi (z_t) \varsigma (\bar{s}_t \bar{u}_t) \right\}. \quad (16)$$

### 2.2.4 Hours Worked in Equilibrium

Hours are determined through negotiation over the joint surplus, i.e. $\max_{h_t} (S_{t}^H)^{1-\vartheta} (S_{t}^F)^{\vartheta}$. Using the expressions for $S_{t}^H$ and $S_{t}^F$ derived above, together with the wage’s splitting rule, the solution to the negotiation problem over hours worked is given by,

$$\frac{dS_{t}^H}{dh_t} = -(1 - \tau^n) \frac{dS_{t}^F}{dh_t},$$
which yields,
\[
\chi^1 = \frac{1}{\lambda_{c,t}} \left( 1 - \tau^n \right) (1 - \alpha)^2 \frac{p_{y,t} y_t}{n_t}. \tag{17}
\]

### 2.3 Production

#### 2.3.1 Intermediate Goods Firms: Optimality Condition for Capital

According to the first order condition with respect to effective capital, the value of the marginal product equals the rental rate,
\[
r_k^t = \alpha \frac{p_{y,t} y_t}{x_t k_t}. \tag{18}
\]

#### 2.3.2 Retailers

There is a continuum of monopolistically competitive retailers indexed by \( i \) on the unit interval. Retailers transform one unit of intermediate goods into one unit of retail goods. The real marginal cost is the relative price \( p_{y,t} \) of intermediate goods. Let \( y_{i,t} \) be the quantity of output produced by retailer \( i \). These goods are aggregated into a tradable good,
\[
y_{r,t} = \int_0^1 \left( y_{i,t} \right)^{\frac{\epsilon-1}{\epsilon}} \, di,
\]
where \( \epsilon > 1 \) is the constant elasticity of demand for each variety. The aggregate tradable good is sold at the nominal price \( P_{r,t} = \left( \int (P_{i,t})^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}} \), where \( P_{i,t} \) is the price of variety \( i \). The demand for \( y_{i,t} \) depends on its relative price and on aggregate demand,
\[
y_{i,t} = \left( \frac{P_{i,t}}{P_{r,t}} \right)^{-\epsilon} y_{r,t}.
\]
Retailers reset prices with a probability \( 1 - \lambda_p \), choosing \( P_{i,t}^* \) to maximize expected real profits,
\[
\Pi_t(i) = \mathbb{E}_t \sum_{s=0}^{\infty} \left( \beta \lambda_p \right)^s \frac{\lambda_{c,t+s}}{\lambda_{c,t}} \left( \left[ \frac{P_{i,t}}{P_{r,t}} - p_{x,t+s} \right] y_{i,t+s} \right),
\]
subject to the demand schedule, where \( P_t \) is the final good price. Since all firms are ex-ante identical (except for the variety they produce), \( P_{i,t}^* = P_{r,t}^* \) for all \( i \). Taking into account \( p_{r,t} \equiv P_{r,t}/P_t \), the resulting expression for the real reset price \( p_{r,t}^* \equiv P_{r,t}^*/P_t \) is,
\[
\frac{p_{r,t}^*}{p_{r,t}} = \frac{\epsilon}{(\epsilon - 1) D_t}, \text{ with } D_t = \frac{\mathcal{N}_t}{N_{t+1}},
\]
where
\[
\mathcal{N}_t = p_{x,t} y_{r,t} + \lambda_p E_t \beta_{t+1} \left( \pi_{r,t+1} \right)^\epsilon N_{t+1},
\]
\[ \mathcal{D}_t = p_{r,t} y_{r,t} + \lambda p \mathcal{E}_t \beta_{t+1} (\pi_{r,t+1})^{1-\epsilon} \mathcal{D}_{t+1}, \]

where \( \pi_{r,t} = P_{r,t}/P_{r,t-1} \) is the producer price inflation. Calvo pricing implies,

\[ (P_{r,t})^{1-\epsilon} = \lambda p (P_{r,t-1})^{1-\epsilon} + (1 - \lambda p) (P^*_{r,t})^{1-\epsilon}. \]

The aggregate tradable good is sold domestically and abroad at quantities \( y_{l,t} \) and \( y^*_{m,t} \),

\[ y_{r,t} = y_{l,t} + y^*_{m,t}; \quad (19) \]

Note that \( y^*_{m,t} \) is the only variable with an asterisk \(*\) that is time dependent.

### 2.3.3 Final Goods Producers

Finally, perfectly competitive firms produce a non-tradable final good \( y_{f,t} \) by aggregating domestic \( y_{l,t} \) and foreign \( y^*_{m,t} \) aggregate retail goods using a CES technology

\[ y_{f,t} = \left[ \left( \varpi \right)^{\frac{1}{\gamma}} (y_{l,t})^\frac{1-\gamma}{\gamma} + (1 - \varpi) \left( y^*_{m,t} \right)^\frac{1-\gamma}{\gamma} \right]^{\gamma}, \quad (20) \]

where \( \varpi \) denotes home bias and \( \gamma \) is the elasticity of substitution. Final good producers maximize profits \( y_{f,t} - p_{r,t} y_{l,t} - e_t p^* r y^*_{m,t} \), where \( p_{r,t} = P_{r,t}/P_t \) and \( p^* r = P^* r / P^* \) denote the real price of \( y_{l,t} \) and \( y^*_{m,t} \), respectively, denominated in each country’s numeraire. We assume the law of one price holds, i.e. \( p_{r,t} = e_t p^* r \). Solving for the optimal demand functions gives

\[ y_{l,t} = \varpi (p_{r,t})^{-\gamma} y_{f,t}, \quad (21) \]

\[ y^*_{m,t} = (1 - \varpi) (e_t p^* r)^{-\gamma} y_{f,t} \quad (22) \]

We substitute out (21) and (22) into (20) to obtain

\[ 1 = \varpi (p_{r,t})^{1-\gamma} + (1 - \varpi) (e_t p^* r)^{1-\gamma}; \quad (23) \]

Then we define implicitly the nominal consumer price index as the value solving (23) for \( P_t \).

### 2.4 Closing the Model

The final output must equal private and public demand (i.e., the government uses final goods to produce public goods and services). Costs related to vacancy posting and search abroad reduce the amount of resources available according to,

\[ y_{f,t} = c_t + i_t + g_t + \kappa u_t + \zeta (\tilde{s}_t \tilde{u}_t) s_t u_t; \quad (24) \]

Aggregating the household budget constraint using the market clearing conditions, the government budget constraint, and aggregate profits, we obtain the law of motion for net foreign
assets,
\[ e_t (r_{f,t-1} b_{f,t-1} - b_{f,t}) = n x_t + e_t \Xi_t , \]  
(25)

where net exports \( n x_t \) are defined as
\[ n x_t \equiv p_{r,t} y_{m,t}^* - e_t p_{r,t}^* y_{m,t} . \]  
(26)

Exports depend on the domestic price divided by the real exchange rate,
\[ y_{m,t}^* = \left( \frac{p_{r,t}}{e_t} \right)^{-\gamma_x} \frac{y_{m,t}^*}{y_{m,t}^*} ; \]  
(27)

where \( \gamma_x \) is the price elasticity and \( \overline{y_{m}}^* \) is the steady-state level of exports, pinned down by the calibrated value of steady-state net foreign assets. Real GDP is defined as,
\[ gdp_t = y_{f,t} + n x_t . \]  
(28)

The nominal exchange rate \( E \) is exogenously set. The nominal interest rate on domestic government bonds \( R_t \) is pinned down endogenously through the Fisher equation,
\[ r_t = \frac{R_t}{E_t \pi_t+1} . \]  
(29)

where consumer price inflation \( \pi_t \) is defined as \( \pi_t = P_t / P_{t-1} \).

3 Quantitative Analysis

3.1 Simulated Shocks

The following table presents information about the shocks used to match the path of consumption and investment in the Eurostat data.

<table>
<thead>
<tr>
<th></th>
<th>t=1</th>
<th>t=2</th>
<th>t=3</th>
<th>t=4</th>
<th>t=5</th>
<th>t=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>risk premium shock</td>
<td>0.15</td>
<td>0.18</td>
<td>0.07</td>
<td>-0.03</td>
<td>-0.025</td>
<td>0.015</td>
</tr>
<tr>
<td>investment efficiency shock</td>
<td>0.06</td>
<td>0.07</td>
<td>0.09</td>
<td>0.04</td>
<td>-0.025</td>
<td>0.25</td>
</tr>
</tbody>
</table>

3.2 Migration Flows for Greece

The figure below depicts annual inflows and outflows of migrants in Greece from 1991.
3.3 More Counterfactuals

Figure 3: Quantitative Analysis: Counterfactual Exercises

(a) The role of labour income tax hikes

Notes: See Figure 4 of the paper.
Figure 4: Quantitative Analysis: Counterfactual Exercises (continued)

(a) The role of cuts in total spending

(b) The role of cuts in productive spending

Notes: See Figure 4 of the paper.
Figure 5: Quantitative Analysis: Counterfactual Exercises (continued)

(a) The role of cuts in utility-enhancing spending

(b) The role of cuts in wasteful spending

Notes: See Figure 4 of the paper.

3.4 Intensive and Extensive Margin

To explore the role of the intensive versus the extensive margin, we modify the utility function as follows,
\[ U(C_t, g_{t}, h_{t}, n_{e,t}) = \Phi^{1-\eta} - \chi \left( \frac{h_t^{1+\xi} n_t + h_e^{1+\xi} n_{e,t}}{1 + \xi} \right) - \Omega (n_{e,t})^{1+\mu} + X \frac{1^{1-\varphi_t}}{1 - \varphi_t}, \]

where \( X > 0 \) is the relative preference for leisure, which is pinned down in steady state by the first-order condition with respect to unemployment (equation (9)), setting in steady state \( l = 1/3 \), and \( \varphi_t \) is the inverse of the Frisch elasticity of labour supply, which takes the standard value 4 in our calibration. The Figure below reports our simulations for the full model (with migration of the unemployed and the employed).

Figure 6: Results of Quantitative Analysis: Intensive and Extensive Margins (Full Model)

Notes: See Figure 4 of the paper.
4 Investment Efficiency Shocks

Figure 7: A Negative Shock to the Marginal Efficiency of Investment

(a) Migration and Labour Market Variables

(b) Output and Monetary Variables

Notes: See Figure 9 of the paper.
5 AR(1) Fiscal Shocks

Figure 8: A Tax Shock Inducing a 1% of GDP Rise in Labour Income Tax Revenue

(a) Migration and Labour Market Variables

(b) Output and Fiscal Variables

Notes: See Figure 6 of the paper.
Figure 9: A 1% Cut in Wasteful Public Spending

(a) Migration and Labour Market Variables

(b) Output and Fiscal Variables

Notes: See Figure 6 of the paper.
6 The Role of Price Stickiness

Figure 10: Labour Tax Hikes: The Role of Price Stickiness

(a) Migration and Labour Market Variables

(b) Output and Fiscal Variables

Notes: We investigate the impact or raising the degree of price rigidities from 0.25 to 0.75. The black line in the Debt/GDP panel reports the path for the debt-to-GDP target. See also Figure 6 of the paper.
Figure 11: (Wasteful) Spending Cuts: The Role of Price Stickiness

(a) Migration and Labour Market Variables

(b) Output and Fiscal Variables

Notes: We investigate the impact or raising the degree of price rigidities from 0.25 to 0.75. The black line in the Debt/GDP panel reports the path for the debt-to-GDP target. Notes: See also Figure 6 of the paper.
7 Fiscal Consolidation Shocks: Additional Results

Figure 12: Comparison in the Model without Migration

(a) Migration and Labour Market Variables

(b) Output and Fiscal Variables

Notes: The black line in the Debt/GDP panel reports the path for the debt-to-GDP target. See also Figure 6 of the paper.
Figure 13: Comparison in the Model with Migration of the Unemployed

(a) Migration and Labour Market Variables

(b) Output and Fiscal Variables

Notes: The black line in the Debt/GDP panel reports the path for the debt-to-GDP target. See also Figure 6 of the paper.
Figure 14: Comparison in the Model with Migration of the Unemployed and the Employed

(a) Migration and Labour Market Variables

(b) Output and Fiscal Variables

Notes: The black line in the Debt/GDP panel reports the path for the debt-to-GDP target. See also Figure 6 of the paper.
Figure 15: Spending Cuts: The Case of Utility-Enhancing Expenditure

(a) Migration and Labour Market Variables

(b) Output and Fiscal Variables

Notes: The black line in the Debt/GDP panel reports the path for the debt-to-GDP target. See also Figure 6 of the paper.
Figure 16: Spending Cuts: The Case of Productive Expenditure

(a) Migration and Labour Market Variables

(b) Output and Fiscal Variables

Notes: The black line in the Debt/GDP panel reports the path for the debt-to-GDP target. See also Figure 6 of the paper.