Roads to Prosperity without Environmental Poverty: The Role of Impatience Supplementary Appendices

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October 13, 2019

Appendix A: Proof of Proposition 1

Existence of multiple equilibria: The method will be to separate function $\Phi(\hat{z}) \equiv Aa(1-\tau)\hat{z}^{a-1} - Ab\tau\hat{z}^{a} - \rho(\bar{N} - \Xi[(1-\tau)A\hat{z}^{a} - b\tau A\hat{z}^{a+1}])$ in two functions and find their intersection to solve it. Let $\Gamma(\hat{z}) \equiv a(1-\tau)\hat{z}^{a-1} - b\tau\hat{z}^{a}$ and $\Lambda(\hat{z}) \equiv \frac{1}{A}\rho(\bar{N} - \Xi[(1-\tau)A\hat{z}^{a} - b\tau A\hat{z}^{a+1}])$ be these two functions. Both $\Gamma(\hat{z})$ and $\Lambda(\hat{z})$ are continuous in $\hat{z}$. First, let us find the domain of z where there cannot be a fixed point. We know that any equilibrium must ensure $\rho(\hat{N}(\hat{z})) = A\Lambda(\hat{z}) \geq 0$. Then it must also hold that $\Gamma(\hat{z}) \geq 0$.

Equation $\Gamma(\hat{z})$ has the following properties:

1. $\Gamma'(\hat{z}) < 0$, $\Gamma''(\hat{z}) > 0$,
1. $\lim_{\hat{z}\to 0}\Gamma(\hat{z}) = +\infty$, $\lim_{\hat{z}\to\frac{a(1-\tau)}{b\tau}}\Gamma(\hat{z}) = 0$.

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From the properties of $\Gamma(\hat{z})$ it follows that it is a strictly decreasing and convex function which starts from $+\infty$ and ends at 0 in the domain of feasible equilibria $(0, \bar{z}]$, $\bar{z} \equiv \frac{a(1-\tau)}{b\tau}$. That is, if there is an equilibrium $\hat{z}$, it will be for $0 < \hat{z} \leq \bar{z}$, because for $\hat{z} > \bar{z}$ then $\Gamma(\hat{z}) < 0$. Also, since $\bar{z} < \frac{1-\tau}{b\tau}$ (because $a < 1$) then for any $z < \bar{z}$ it follows that $\hat{\omega} > 0$ (from $\hat{\omega}(\hat{z}) = A(1-\tau)\hat{z}^{a-1} - Ab\tau\hat{z}^{a}$).

Equation $\Lambda(\hat{z})$ has the following properties:

1. $\Lambda(\hat{z})$ has a maximum at $z_{max} = \frac{1}{1+a}\bar{z}$,
2. $\lim_{\hat{z}\to 0}\Lambda(\hat{z}) = \lim_{\hat{z}\to\frac{1-\tau}{b\tau}}\Lambda(\hat{z}) = \rho(\bar{N})/A$.

Since $\rho'(N) < 0$, we have $\Lambda'(\hat{z}) > 0$ for $a(1-\tau)\hat{z}^{a-1} - b(1+a)\tau\hat{z}^{a} > 0 \implies \hat{z} < \frac{1}{1+a}\bar{z}$ and $\Lambda'(\hat{z}) < 0$ for $\hat{z} > \frac{1}{1+a}\bar{z}$. Thus, $\Lambda(\hat{z})$ has a maximum at $z_{\max} = \frac{1}{1+a}\bar{z}$. Note also that $z_{\max} < \bar{z}$ as $a > 0$. Additionally, since Λ starts and ends at $\rho(\bar{N})/A$ it follows that it is an inverse U-shaped curve with $z_{max} \in (0, \bar{z})$. With regards to $\rho(\bar{N})$ we have the following options: $\rho(\bar{N}) \geq 0$ and $\rho(\bar{N}) < 0$. If $\rho(\bar{N}) \geq 0$, since $\bar{z} < \frac{1-\tau}{b\tau}$ and $\lim_{\hat{z}\to 0}\Gamma(\hat{z}) = +\infty$, Λ crosses Γ once and there is a unique equilibrium. For $\rho(\bar{N}) < 0$ and since $\Lambda(\hat{z})$ is an inverse U-shaped function, Λ crosses Γ at most twice.

Therefore, the sufficient conditions for two equilibria are $\rho(\bar{N}) < 0$ and $\Lambda(z_{max}) > \Gamma(z_{max})$, the latter excluding the possibility of no equilibrium, or the trivial case of the single, tangency, equilibrium. A graphical illustration is provided in Figure 1 of this Appendix.

Ranking of Equilibria: Let $\hat{z}_1 < \hat{z}_2$ be the two equilibria for which $\Phi(\hat{z}_1) = \Phi(\hat{z}_2) = 0$. To find the ranking of the correspoding $\hat{\omega}_1$ and $\hat{\omega}_2$ note that $\hat{\omega}'(\hat{z}) = (a-1)(1-\tau)A\hat{z}^{a-2} - abA\tau\hat{z}^{a-1} < 0$. Thus, $\hat{\omega}$ is a strictly decreasing function of $\hat{z}$, $\hat{z}_1 < \hat{z}_2 \Longrightarrow \hat{\omega}_1 > \hat{\omega}_2$. In the same way $g'(\hat{z}) = b\tau a\hat{z}^{a-1} > 0$, so $\hat{z}_1 < \hat{z}_2 \Longrightarrow g_1 < g_2$. The ranking for the rate of time preference comes from the analysis above. Then, $\hat{z}_1 < \hat{z}_2 \Longrightarrow \Lambda(\hat{z}_1) > \Lambda(\hat{z}_2) \implies \rho_1 > \rho_2$. Also for $\hat{z}_1 < \hat{z}_2 \Rightarrow \hat{\omega}_1 > \hat{\omega}_2$ and environmental quality, $N_1 < N_2$. So, in case of two balanced growth rates with low growth, g_1, and high growth, g_2, the endogenous variables are ranked as $\rho_1 > \rho_2$, $\hat{\omega}_1 > \hat{\omega}_2$, $\hat{z}_1 < \hat{z}_2$, $N_1 < N_2$.

Stability analysis: To examine stability, we compute the Jacobian Matrix of the three-dimensional dynamical system (13)-(15) of the main text. Around a steady state $\{\hat{\omega}, \hat{z}, \hat{N}\}$

we get the linearized version of the dynamic system as:

$$\begin{bmatrix} \dot{\omega} \\ \dot{z} \\ \dot{N} \end{bmatrix} = J \begin{bmatrix} \omega - \hat{\omega} \\ z - \hat{z} \\ N - \hat{N} \end{bmatrix},$$

where $J \equiv \begin{bmatrix} J_{\omega\omega} & J_{\omega z} & J_{\omega N} \\ J_{z\omega} & J_{zz} & J_{zN} \\ J_{N\omega} & J_{Nz} & J_{NN} \end{bmatrix}$ the Jacobian evaluated at the steady state. The elements of the matrix are given by:

$J_{\omega\omega} = \hat{\omega},\ J_{\omega z} = A(1-\tau)(a-1)^2\hat{z}^{a-2}\hat{\omega},\ J_{\omega N} = -\rho'(\hat{N})\hat{\omega},$

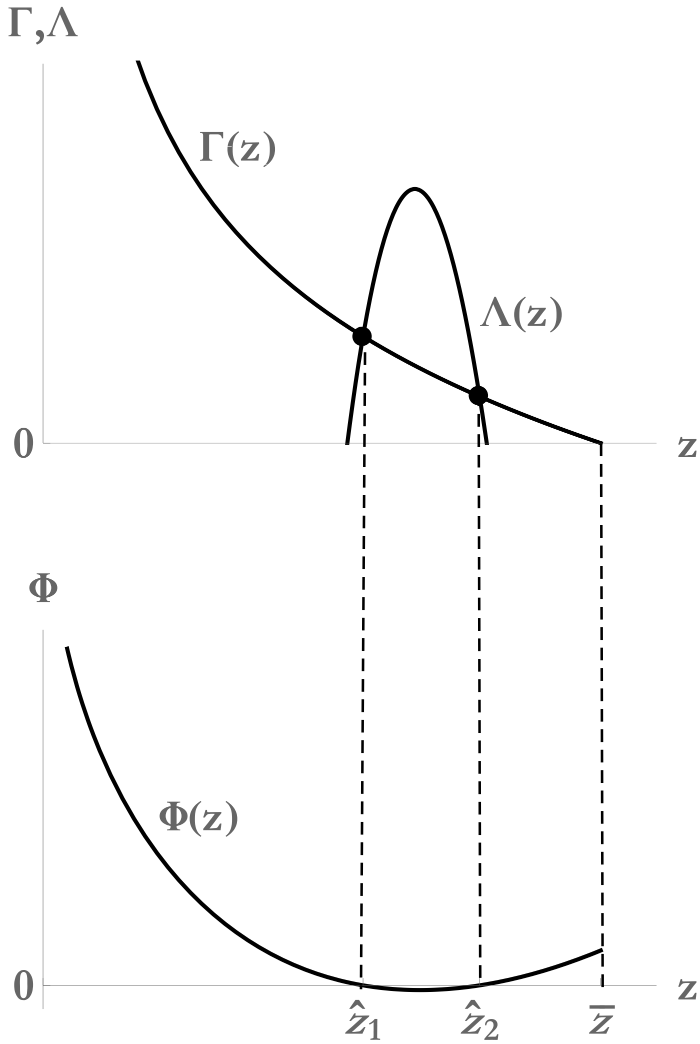
$J_{z\omega} = -\hat{z},\ J_{zz} = -\frac{A}{\Xi}\frac{\Lambda'(\hat{z})}{\rho'(\hat{N})} - \hat{\omega},\ J_{zN} = 0,$

$J_{N\omega} = -\Xi\hat{z}, J_{Nz} = -\Xi\hat{\omega},\ J_{NN} = -(1-\delta_N).$

The local dynamics of the economy are nontrivial and analytically intractable. For that reason we resort to numerical simulations, to (i) compute the two equilibria, (ii) solve for the characteristic equation and compute the eigenvalues of J for those equilibria.

In particular, assuming $\rho(N) = \bar{\rho} - \gamma N$ and using the set of parameter values used in the paper ($\alpha = 0.5$, $A = 0.659$, $\delta = 0.14$, $\tau = 0.561$, $b = 0.751$, $\delta_N = 0.9$, $\xi = 0.4$, $s = 1$, $\bar{N} = 20$, $\gamma = 1$, $\bar{\rho} = 0.2$), we obtain for the low-growth and bad-environment equilibrium ($g_1 = 0.004$, $\hat{N}_1 = 0.066$) the following eigenvalues: $\varepsilon_1 = -0.054 + 1.380i$, $\varepsilon_2 = -0.054 - 1.380i$, $\varepsilon_3 = 0.069$, and for the high-growth and good-environment equilibrium ($g_2 = 0.044$, $\hat{N}_2 = 0.166$) the following eigenvalues: $\varepsilon_1 = -0.051 + 1.389i$, $\varepsilon_2 = -0.051 - 1.389i, \varepsilon_3 = -0.058$. Given that there exist two predetermined and one jump variables in our dynamical system, it follows that both equilibria are stable. For the good equilibrium this is straightforward since all real parts are negative. For the bad equilibrium this follows from the fact that the complex eigenvalues are dominant over the third one and have negative real parts. Thus, there exist parameter values such that the multiple (two) equilibria derived in Proposition 1 are both stable and, hence, meaningful for examining changes in the structural and policy parameters of the model.

Figure 1: **Multiple equilibria**



Appendix B: Proof of Proposition 2

The task is to calculate the effect of an increase of A on the steady state variables $\hat{\omega}, \hat{z}, \hat{N}$ and subsequently on $\rho(\hat{N})$ and $g(\hat{N})$, in the presence of multiple equilibria.

To ease exposition we repeat here that for the two equilibria with $\hat{z}_1 < \hat{z}_2 < \bar{z}$ that solve $\Phi(\hat{z}) \equiv \Gamma(\hat{z}) - \Lambda(\hat{z}) = 0$, with $\Gamma(\hat{z}) \equiv b\tau\hat{z}^{a-1}(\bar{z} - \hat{z})$ and $\Lambda(\hat{z}) \equiv \frac{1}{A}\rho\left(\bar{N} - \Xi Ab\tau\hat{z}^{a}\left(\frac{1-\tau}{b\tau} - \hat{z}\right)\right)$ holds $\hat{z}_1 < z_{max}$ for the bad equilibrium, and for the good $\hat{z}_2 > z_{max}$. Moreover, it follows from the above and $\rho'(\cdot) < 0$ that $\hat{z} < (>) z_{max} \implies \Lambda'(\hat{z}) > (<) 0$, while $\Gamma'(\hat{z}) < 0$; see also Figure 1 of Appendix A. Additionally, it follows from eq. (14) of the main text that $\partial\hat{\omega}/\partial\hat{z} < 0$.

The effect of a productivity increase on functions $\Gamma(\hat{z})$ and $\Lambda(\hat{z})$ is given by:

1. $\frac{\partial\Gamma(\hat{z})}{\partial A} = 0$, i.e. function $\Gamma(\hat{z})$ does not shift with A,
2. $\frac{\partial\Lambda(\hat{z})}{\partial A} = -\frac{1}{A^2}\rho(\hat{N})\left[1 + \epsilon_{\rho N}\left(\frac{\bar{N}}{\hat{N}} - 1\right)\right]$, with $\epsilon_{\rho N} \equiv \rho'(\hat{N})\hat{N}/\rho(\hat{N}) < 0$.

We get from 2. above that if $\epsilon_{\rho N} < -(\bar{N}/\hat{N} - 1)^{-1} < 0 \implies \partial\Lambda(\hat{z})/\partial A > 0$, i.e. the function shifts up. Provided that the subjective discount rate ρ responds sufficiently to a change in environmental quality, as measured by the elasticity $\epsilon_{\rho N}$, function $\Lambda(\hat{z})$ shifts up and since $\Gamma(\hat{z})$ stays unchanged the two equilibria z_1 and z_2 move further apart from each other, i.e. $\partial z_1/\partial A < 0$ and $\partial z_2/\partial A > 0$; see Figure 2 of this Appendix.

With regards to the consumption-capital ratio, we can write from eq. (14) of the main text $\frac{\partial\hat{\omega}}{\partial A} = \frac{\hat{\omega}}{A} + \frac{\partial\hat{\omega}}{\partial\hat{z}}\frac{\partial\hat{z}}{\partial A}$. For the bad equilibrium we have $\partial z_1/\partial A < 0$, while $\partial\hat{\omega}/\partial\hat{z} < 0 \implies \partial\omega_1/\partial A > 0$. From eq. (13) we get that $\frac{\partial\omega_1}{\partial A} = \rho'(\cdot)\frac{\partial N_1}{\partial A} + (1-a)(1-\tau)z_1^{a-1} - A(1-a)^2(1-\tau)z_1^{a-2}\frac{\partial z_1}{\partial A}$. Since $\partial\omega_1/\partial A > 0, \rho'(\cdot) < 0, \partial z_1/\partial A < 0 \implies \partial N_1/\partial A < 0$. To study the effect of an increase in productivity on growth for the bad equilibrium, notice that, from eq. (6) of the main text, $\frac{\partial g}{\partial A} = \frac{\partial r(\hat{z})}{\partial A} - \rho'(\cdot)\frac{\partial\hat{N}(\hat{z})}{\partial A}$. Using eq. (2), (6) and the definition of $\epsilon_{\rho N}$ from above, we get that $\frac{\partial g}{\partial A} = \frac{g+\delta}{A} + \frac{\rho(\cdot)}{A}\left[1 + \epsilon_{\rho N}\left(\frac{\bar{N}}{\hat{N}} - 1\right)\right] + \left[-a(1-a)(1-\tau)\hat{z}^{a-1} + \rho'(\cdot)\Xi b\tau(1+a)\hat{z}^{a}(z_{max} - \hat{z})\right]\frac{\partial\hat{z}}{\partial A}\frac{A}{\hat{z}}$. Since $\partial z_1/\partial A < 0$ and $z_1 < z_{max}$ for the bad equilibrium, the previous condition of $\epsilon_{\rho N} < -(\bar{N}/\hat{N} - 1)^{-1} < 0$ is a necessary condition for $\partial g_1/\partial A < 0$. Accordingly, if the responce of the RTP to a change in environmental quality is sufficiently high to ensure $\frac{\partial r(\hat{z})}{\partial A} < -\rho'(\cdot)\frac{\partial\hat{N}(\hat{z})}{\partial A}$, then $\partial g_1/\partial A < 0$.

Finally, as illustrated in Figure 2 of the main text, there exist parameter values such that a small increase in productivity from $A = 0.659$ to $A = 0.66$, reduces g_1, N_1, z_1, while increases ω_1 and $\rho(N_1)$. The opposite holds true for the variables in the good equilibrium.

Figure 2: **Effect of productivity increase on $\hat{z}$**