Fiscal Austerity and Migration: A Missing Link

Online Appendix
(not intended for publication)

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1 First order conditions of the household problem

The household's Lagrangean can be written as

$$
\begin{aligned}
\mathcal{L} = \quad & \sum_{t=0}^{\infty} \quad \beta^t \left\{ \frac{\left(C_t - \zeta \tilde{C}_t\right)^{1-\eta}}{1-\eta} - \chi \frac{h_t^{1+\xi} n_t + h_e^{1+\xi} n_{e,t}}{1+\xi} - \Omega \frac{(n_{e,t})^{1+\mu}}{1+\mu} \right. \\
& -\lambda_{c,t} \quad \Big[(1+\tau^c) c_t + i_t + b_{g,t} + e_t r_{f,t-1} b_{f,t-1} + \phi(z_t) n_t + \varsigma(\tilde{s}_t \tilde{u}_t) s_t u_t - (1-\tau^n) w_t h_t n_t - \mathrm{b} u_t \\
& \qquad - \left[r_t^k - \tau^k \left(r_t^k - \delta_t \right) \right] x_t k_t - r_{t-1} b_{g,t-1} - e_t b_{f,t} - \Pi_t^p - T_t \\
& \qquad + e_t \left((1+\tau^{c\star}) c_{e,t} - (1-\tau^{n\star}) w^\star n_{e,t} \right) \Big] \\
& -\lambda_{u,t} \quad (n_t + n_{e,t} + u_t - \overline{n}) \\
& -\lambda_{k,t} \quad \left[k_{t+1} - \left[1 - \frac{\omega}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t - \left(1 - \bar{\delta} x_t^{\iota} \right) k_t \right] \\
& -\lambda_{n,t} \quad \left[n_{t+1} - (1 - \sigma - \psi_H^\star \varphi(z_t)) n_t - \psi_{H,t} (1 - s_t) u_t \right] \\
& -\lambda_{e,t} \quad \left. \left[n_{e,t+1} - (1-\sigma^\star) n_{e,t} - \psi_H^\star \left(s_t u_t + \varphi(z_t) n_t \right) \right] \right\}.
\end{aligned}
$$

We assume external habits in consumption, meaning that $\tilde{C}_t \equiv C_{t-1}$ is taken as given in period t. The choice variables comprise c_t, k_{t+1}, i_t, x_t, $b_{g,t}$, $b_{f,t}$, n_{t+1}, $n_{e,t+1}$, u_t, s_t, and z_t. The corresponding first order conditions are the following:

c_t :

$$\lambda_{c,t} (1+\tau^c) = (C_t - \zeta C_{t-1})^{-\eta} \tag{1}$$

k_{t+1} :

$$\lambda_{k,t} = \beta \lambda_{c,t+1} \left(\left[r_{t+1}^k - \tau^k \left(r_{t+1}^k - \delta_{t+1} \right) \right] x_{t+1} \right) + \beta \lambda_{k,t+1} (1 - \delta_{t+1}) \tag{2}$$

i_t :

$$\lambda_{c,t} - \lambda_{k,t} \left\{ 1 - \frac{\omega}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \omega \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right\} = \beta \lambda_{k,t+1} \omega \left(\frac{i_{t+1}}{i_t} - 1 \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \tag{3}$$

x_t :

$$\lambda_{k,t}\iota\bar{\delta}\left(x_t\right)^{\iota-1} = \lambda_{c,t}\left\{r_t^k - \tau^k\left(r_t^k - (1+\iota)\,\delta_t\right)\right\} \tag{4}$$

$b_{g,t+1}$:

$$1 = \beta\frac{\lambda_{c,t+1}}{\lambda_{c,t}}r_t \tag{5}$$

$b_{f,t+1}$:

$$1 = \beta\frac{\lambda_{c,t+1}}{\lambda_{c,t}}\frac{e_{t+1}}{e_t}r_{f,t} \tag{6}$$

n_{t+1} :

$$\begin{aligned}\lambda_{n,t}/\beta &= \lambda_{c,t+1}\left[\left(1-\tau^n\right)w_{t+1}h_{t+1} - \phi\left(z_{t+1}\right)\right] - \lambda_{u,t+1} - \chi\frac{h_{t+1}^{1+\xi}}{1+\xi} \\ &\quad +\lambda_{n,t+1}\left(1-\sigma-\psi_H^{\star}\varphi\left(z_{t+1}\right)\right) + \lambda_{e,t+1}\psi_H^{\star}\varphi\left(z_{t+1}\right)\end{aligned} \tag{7}$$

$n_{e,t+1}$:

$$\begin{aligned}\lambda_{e,t}/\beta &= \lambda_{c,t+1}\left(1-\tau^{n\star}\right)e_{t+1}w^{\star}h_e - \chi\frac{h_e^{1+\xi}}{1+\xi} - \Omega\left(n_{e,t+1}\right)^{\mu} - \lambda_{u,t+1} \\ &\quad +\lambda_{e,t+1}\left(1-\sigma^{\star}\right)\end{aligned} \tag{8}$$

u_t :

$$\lambda_{u,t} = \lambda_{c,t}\left(\mathrm{b} - \varsigma\left(\tilde{s}_t\tilde{u}_t\right)s_t\right) + \lambda_{n,t}\psi_{H,t}\left(1-s_t\right) + \lambda_{e,t}\psi_H^{\star}s_t \tag{9}$$

s_t :

$$\lambda_{e,t}\psi_H^{\star} - \lambda_{c,t}\varsigma\left(\tilde{s}_t\tilde{u}_t\right) = \lambda_{n,t}\psi_{H,t} \tag{10}$$

z_t :

$$\lambda_{c,t}\frac{\phi'\left(z_t\right)}{\varphi'\left(z_t\right)} = \psi_H^{\star}\left(\lambda_{e,t} - \lambda_{n,t}\right) \tag{11}$$

To derive the exact form of equations (13) and (14) in the paper, we proceed as follows.

Combining equations (9) and (10) we get the following two expressions

$$\lambda_{u,t} = \lambda_{c,t}\mathrm{b} + \lambda_{n,t}\psi_{H,t}\,, \tag{12}$$

$$\lambda_{u,t} = \lambda_{c,t}\left(\mathrm{b} - \varsigma\left(\tilde{s}_t\tilde{u}_t\right)\right) + \lambda_{e,t}\psi_H^{\star}\,. \tag{13}$$

We then use (12) to replace $\lambda_{u,t+1}$ in (7)

$$\begin{aligned}\lambda_{n,t}/\beta &= \lambda_{c,t+1}\left[\left(1-\tau^n\right)w_{t+1}h_{t+1} - \mathrm{b} - \phi\left(z_{t+1}\right)\right] - \chi\frac{h_{t+1}^{1+\xi}}{1+\xi} \\ &\quad + \lambda_{n,t+1}\left(1-\sigma-\psi_{H,t+1}-\psi_H^{\star}\varphi\left(z_{t+1}\right)\right) + \lambda_{e,t+1}\psi_H^{\star}\varphi\left(z_{t+1}\right)\,.\end{aligned} \tag{14}$$

and we use (13) to replace $\lambda_{u,t+1}$ in (8)

$$\begin{aligned}\lambda_{e,t}/\beta &= \lambda_{c,t+1}\left(\left(1-\tau^{n\star}\right)e_{t+1}w^{\star}h_e - \mathrm{b} + \varsigma\left(\tilde{s}_{t+1}\tilde{u}_{t+1}\right)\right) - \chi\frac{h_e^{1+\xi}}{1+\xi} - \Omega\left(n_{e,t+1}\right)^{\mu} \\ &\quad + \lambda_{e,t+1}\left(1-\sigma^{\star}-\psi_H^{\star}\right)\,.\end{aligned} \tag{15}$$

The last two expressions correspond to equations (13) and (14) in the paper.

2 The wage-hours bargaining problem

2.1 The surplus for the household

The surplus for workers consists of the asset value of employment net of the outside option (value of being unemployed): $S_t^H \equiv V_t^{E_H} - V_t^{U_H}$. The asset value of employment, V^{E_H}, is given by

$$\begin{aligned}V_t^{E_H} &\equiv \left(1-\tau^n\right)w_t h_t - \phi\left(z_t\right) - \frac{\chi}{\lambda_{c,t}}\frac{h_t^{1+\xi}}{1+\xi} \\ &\quad + \mathrm{E}_t\beta_{t+1}\left\{\left(1-\sigma-\psi_H^{\star}\varphi\left(z_t\right)\right)V_{t+1}^{E_H} + \sigma V_{t+1}^{U_H} + \psi_H^{\star}\varphi\left(z_t\right)V_{t+1}^{E_F}\right\}\,,\end{aligned}$$

where V^{U_H}, the value of being unemployed at Home, is given by

$$V_t^{U_H} \equiv \mathrm{b} + \mathrm{E}_t\beta_{t+1}\left\{\psi_{H,t}V_{t+1}^{E_H} + \left(1-\psi_{H,t}\right)V_{t+1}^{U_H}\right\}\,.$$

Hence, the worker's surplus, S_t^H, is given by

$$\begin{aligned} S_t^H &= (1-\tau^n) w_t h_t - \phi(z_t) - \mathrm{b} - \frac{\chi}{\lambda_{c,t}} \frac{h_t^{1+\xi}}{1+\xi} + \mathrm{E}_t \beta_{t+1} (1-\sigma-\psi_{H,t}) S_{t+1}^H \\ &+ \mathrm{E}_t \beta_{t+1} \psi_H^\star \varphi(z_t) \left\{ V_{t+1}^{E_F} - V_{t+1}^{E_H} \right\} . \end{aligned}$$

In the previous expression, $V_t^{E_F}$ marks the value of being employed abroad, and is given by

$$\begin{aligned} V_t^{E_F} &\equiv e_t (1-\tau^{n\star}) w^\star h_e - \frac{\chi}{\lambda_{c,t}} \frac{h_e^{1+\xi}}{1+\xi} - \frac{\Omega}{\lambda_{c,t}} (n_{e,t})^\mu \\ &+ \mathrm{E}_t \beta_{t+1} \left\{ (1-\sigma^\star) V_{t+1}^{E_F} + \sigma^\star V_{t+1}^{U_F} \right\} , \end{aligned}$$

where we assume that when an emigrant looses her job she joins the stock of job-seekers searching for a job abroad. The value of job-seeking abroad, $V_t^{U_F}$, is given by

$$V_t^{U_F} \equiv \mathrm{b} - \varsigma(\tilde{s}_t \tilde{u}_t) + \mathrm{E}_t \beta_{t+1} \left\{ \psi_H^\star V_{t+1}^{E_F} + (1-\psi_H^\star) V_{t+1}^{U_F} \right\} .$$

Hence, migrant workers' surplus, $S_{h,t}^F \equiv V_t^{E_F} - V_t^{U_F}$, is given by

$$\begin{aligned} S_{h,t}^F &= e_t (1-\tau^{n\star}) w^\star h_e - \mathrm{b} - \varsigma(\tilde{s}_t \tilde{u}_t) - \frac{\chi}{\lambda_{c,t}} \frac{h_e^{1+\xi}}{1+\xi} - \frac{\Omega}{\lambda_{c,t}} (n_{e,t})^\mu \\ &+ (1-\sigma^\star - \psi_H^\star) \mathrm{E}_t \beta_{t+1} S_{h,t+1}^F . \end{aligned}$$

Optimality implies that the value of jobseeking at home or abroad must be equal (see the FOC with respect to u_t in the household's problem). Hence, $V_t^{U_H} = V_t^{U_F}$ implies:

$$\psi_{H,t} \mathrm{E}_t \beta_{t+1} S_{t+1}^H = \psi_H^\star \mathrm{E}_t \beta_{t+1} S_{h,t+1}^F - \varsigma(\tilde{s}_t \tilde{u}_t) .$$

2.2 The surplus for the firm

For the firm, the surplus from a match is given by

$$S_t^F = p_{x,t} (1-\alpha) \frac{y_t}{n_t} - w_t h_t + (1-\sigma-\psi_H^\star \varphi(z_t)) \mathrm{E}_t \beta_{t+1} S_{t+1}^F ,$$

which, using the FOC wrt υ_t, can be written as

$$S_t^F = (1-\alpha) \frac{p_{x,t} y_t}{n_t} - w_t h_t + (1-\sigma-\psi_H^\star \varphi(z_t)) \frac{\kappa}{\psi_{F,t}} .$$

2.3 The Nash-bargained wage

Inserting the two surpluses into the splitting rule $(1-\vartheta)(1-\tau^n)S_t^F = \vartheta S_t^H$ and solving for the wage yields

$$\begin{aligned} w_t h_t &= (1-\vartheta)\left\{(1-\alpha)\frac{p_{x,t}y_t}{n_t} + (1-\varphi(z_t))\frac{\psi_{H,t}}{\psi_{F,t}}\kappa\right\} \\ &+\frac{\vartheta}{(1-\tau^n)}\left\{\mathrm{b} + \frac{\chi}{\lambda_{c,t}}\frac{h_t^{1+\xi}}{1+\xi} + \phi(z_t) - \varphi(z_t)\varsigma(\tilde{s}_t\tilde{u}_t)\right\}. \end{aligned}$$

2.4 Hours worked in equilibrium

Hours are determined through negotiation over the joint surplus of workers and firms

$$\max_{h_t} \quad \left(S_t^H\right)^{1-\vartheta}\left(S_t^F\right)^{\vartheta}.$$

Using the expresions for S_t^H and S_t^F derived above, together with the wage's splitting rule, the solution to the negotiation problem over hours worked is given by

$$\frac{dS_t^H}{dh_t} = -(1-\tau^n)\frac{dS_t^F}{dh_t},$$

which yields:

$$\chi\frac{h_t^{1+\xi}}{\lambda_{c,t}} = (1-\tau^n)(1-\alpha)^2\frac{p_{x,t}y_t}{n_t}.$$

3 Simulations

The following table presents information about the shocks used in the simulation exercise.

Table 1: Simulated shocks

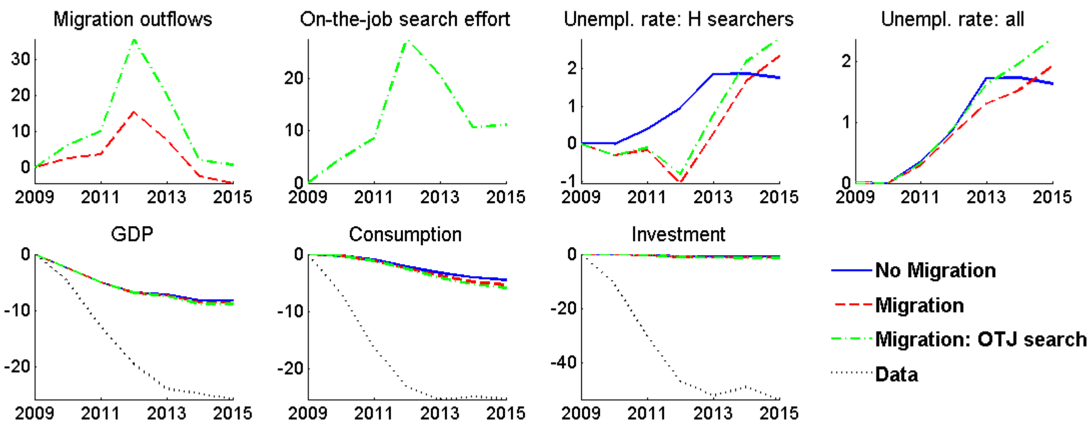
Shocks:	t	1	2	3	4	5	6
risk premium		0.15	0.18	0.07	-0.03	-0.025	0.015
investment specific		0.06	0.07	0.09	0.04	-0.025	0.25

These are the shocks used to match the path of consumption and investment in the data. The parameter γ_x, the price elasticity of exports, is used to approximate the response of GDP in the model to the data. Data is from the Eurostat.

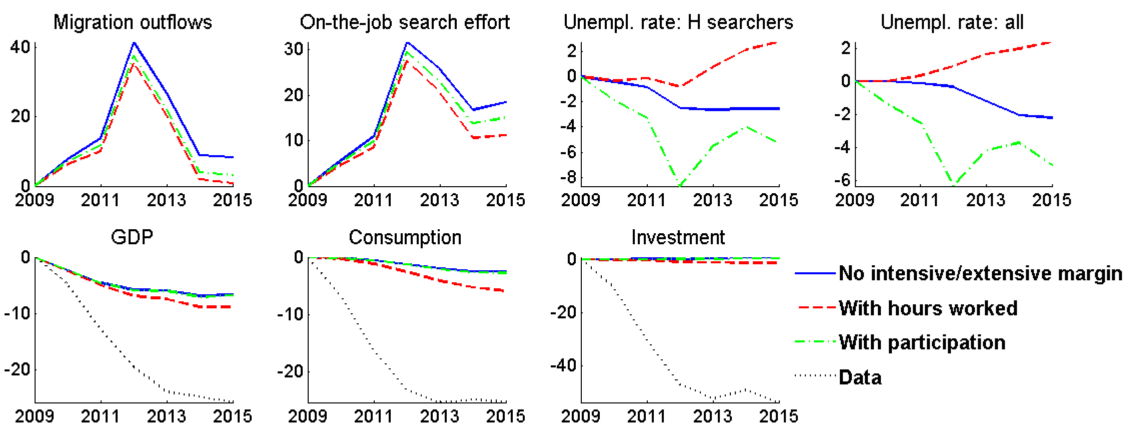
4 Additional figures

Figure 1: Fiscal consolidation mix in Greece: no risk premium and investment shocks

(a) comparison of the main specifications



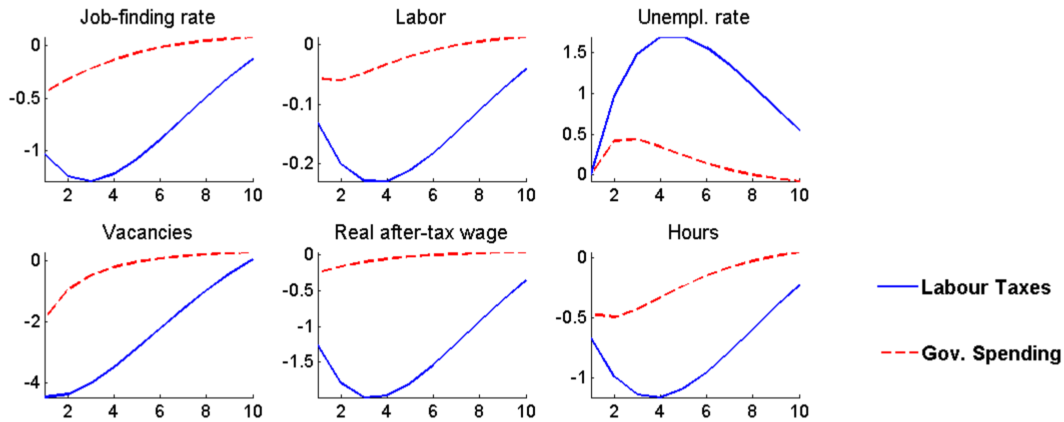
(b) intensive and extensive margin



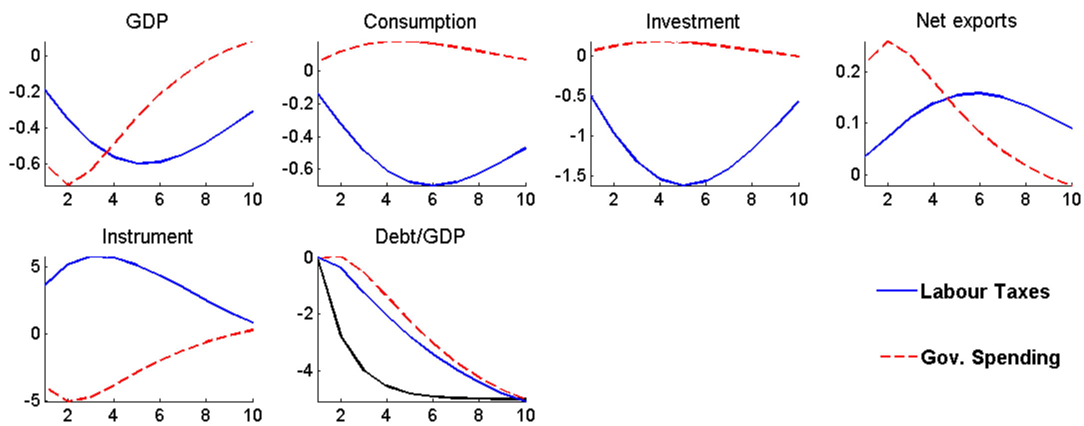
Responses for migration outflows are in levels (thousands persons). All other responses are in percent deviations from steady state. Consumption refers to consumption of the domestic good. OTJ denotes on the job. Unempl. rate: all and Unempl. rate: H searchers denote measures of the unemployment rate including and excluding, respectively, the share of unemployed that look for a job abroad.

Figure 2: Comparison of instruments without labour force mobility

(a) Migration and Labour Market



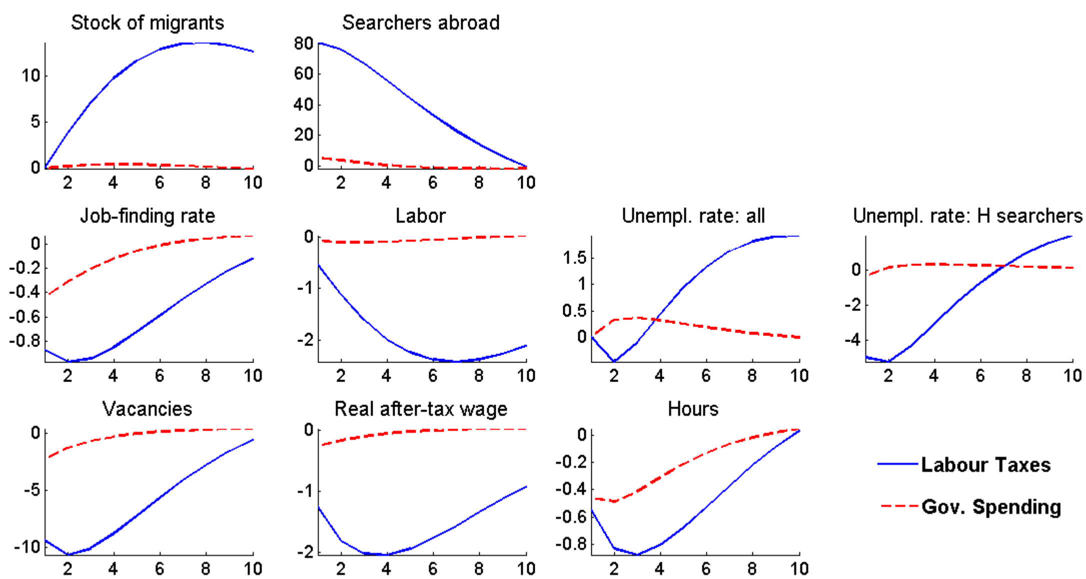
(b) Aggregates



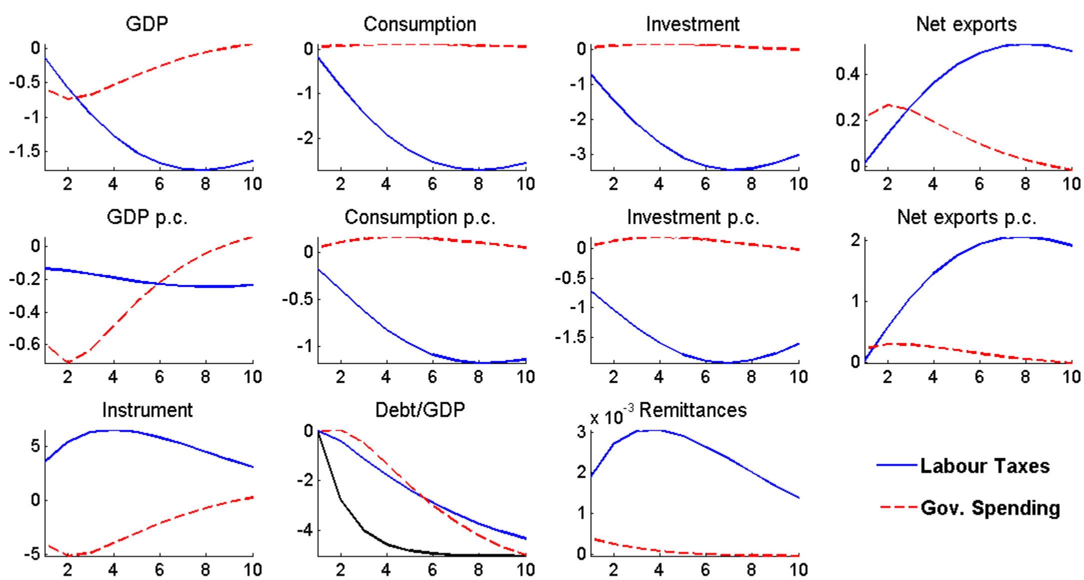
Responses for the job-finding rate and net exports are in levels. All other responses are in percent deviations from steady state. Consumption refers to consumption of the domestic good. The black line in the Debt/GDP panel reports the path for the debt-to-GDP target.

Figure 3: Comparison of instruments with search abroad of the unemployed

(a) Migration and Labour Market



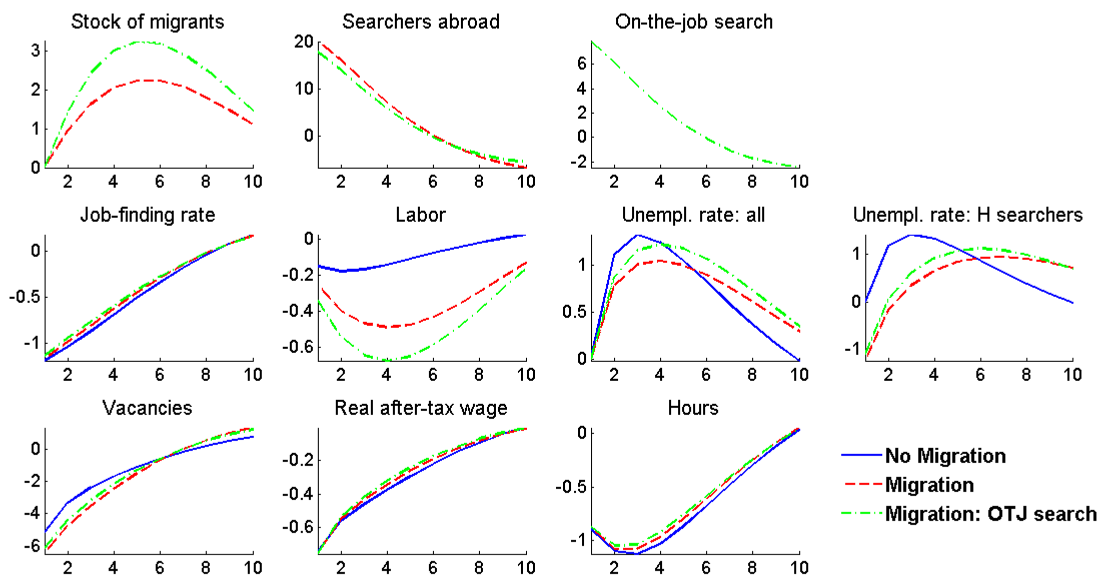
(b) Aggregates



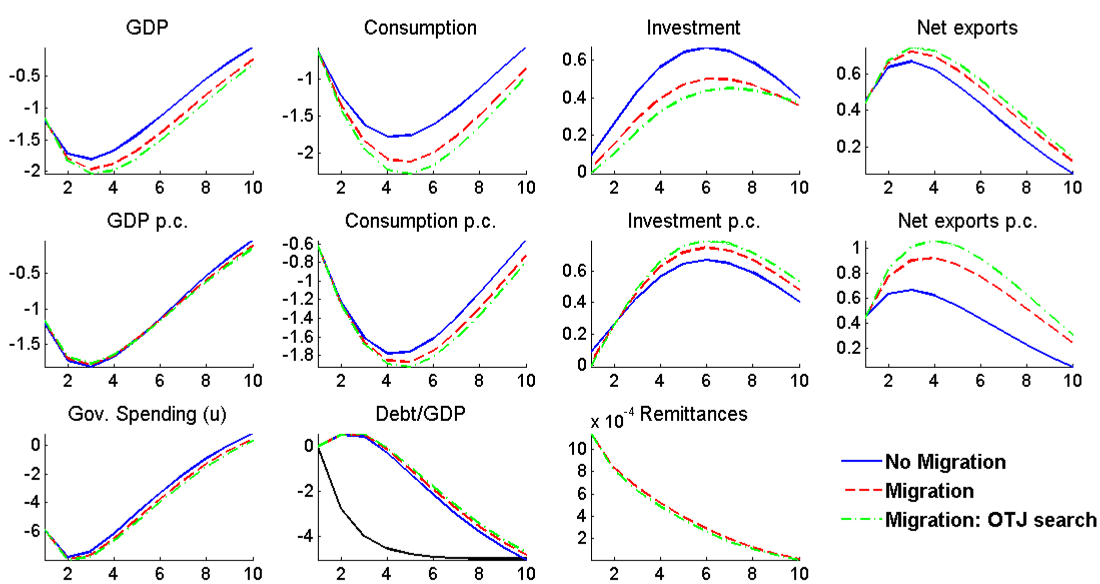
Responses for the job-finding rate and net exports are in levels. All other responses are in percent deviations from steady state. Consumption refers to consumption of the domestic good. p.c. denotes per capita. Unempl. rate: all and Unempl. rate: H searchers denote measures of the unemployment rate including and excluding, respectively, the share of unemployed that look for a job abroad. The black line in the Debt/GDP panel reports the path for the debt-to-GDP target.

Figure 4: Spending-based consolidation: utility-enhancing public expenditure

(a) Migration and Labour Market



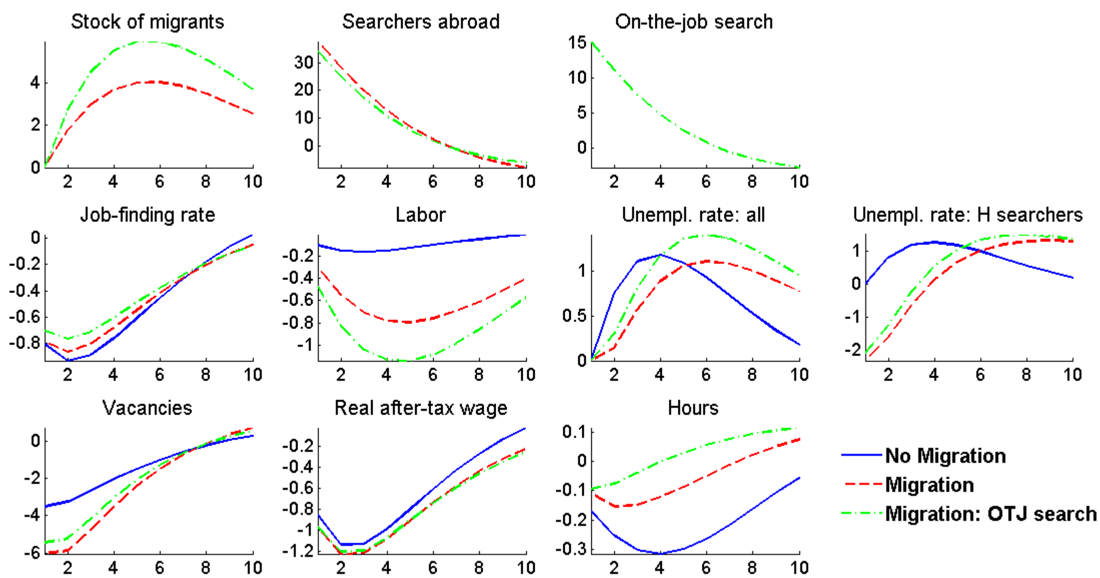
(b) Aggregates



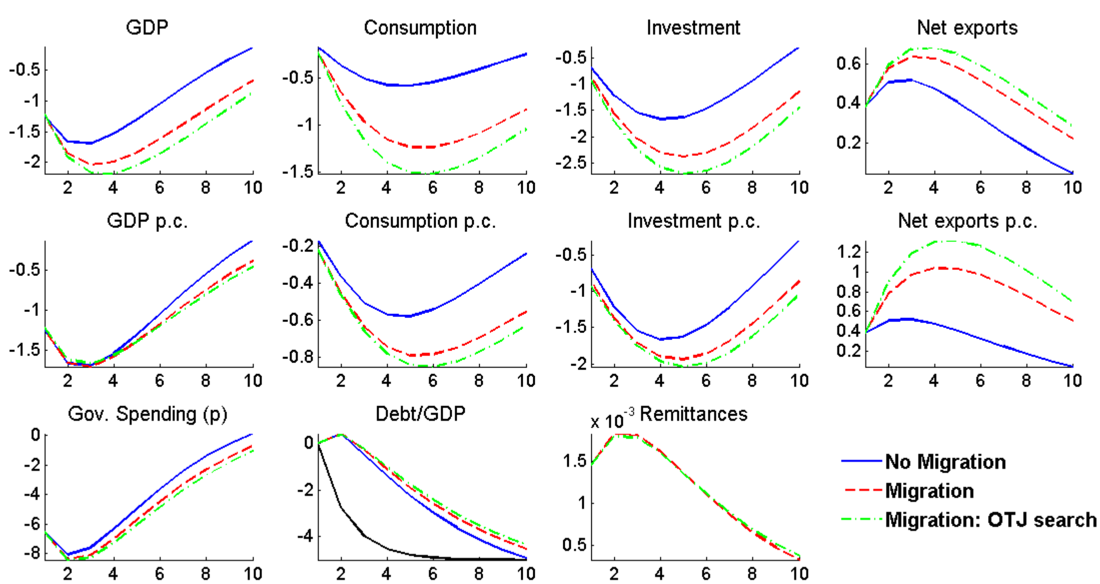
Responses for the job-finding rate and net exports are in levels. All other responses are in percent deviations from steady state. Consumption refers to consumption of the domestic good. OTJ denotes on the job and p.c. denotes per capita. Unempl. rate: all and Unempl. rate: H searchers denote measures of the unemployment rate including and excluding, respectively, the share of unemployed that look for a job abroad. The black line in the Debt/GDP panel reports the path for the debt-to-GDP target.

Figure 5: Spending-based consolidation: productive public expenditure

(a) Migration and Labour Market



(b) Aggregates



Responses for the job-finding rate and net exports are in levels. All other responses are in percent deviations from steady state. Consumption refers to consumption of the domestic good. OTJ denotes on the job and p.c. denotes per capita. Unempl. rate: all and Unempl. rate: H searchers denote measures of the unemployment rate including and excluding, respectively, the share of unemployed that look for a job abroad. The black line in the Debt/GDP panel reports the path for the debt-to-GDP target.