Roads to Prosperity without Environmental Poverty: The Role of Impatience and Fiscal Policy

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Abstract

This paper advances the hypothesis that impatience negatively depends on environmental quality and, thereby, aims to explain why some countries stagnate in an 'environmental and economic poverty trap'. In particular, for low levels of environmental quality, advancements in productivity lead impatient agents to direct income increases to consumption (rather than savings), depleting further the environment. Given that productivity increases do not help such economies to escape the trap (contrary to perceived notions), we show how an escape is possible via a behavioral change brought about by fiscal policy. However, for high levels of environmental quality, pursuit of the latter leads to higher growth at the expense of environmental quality under productivity advancements. This occurs despite the endogenous shift of public spending from infrastructure to the environment and can help us also explain why some advanced economies may not succeed in cleaning the environment effectively.

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1 Introduction

Higher economic growth is widely perceived to result in greater environmental degradation through higher pollution in the early stages of development, which tends to get reversed beyond a point. Following Grossman and Krueger (1995), this inverse U-shaped relation between growth and pollution is commonly refered to as the Environmental Kuznets Curve (EKC). However, few studies in the EKC literature actually map the historical movement of a country along an EKC (Liu, 2012). For instance, developing countries are often stuck in 'environmental and economic poverty traps' characterized by environmental degradation and low growth (Fact 1), without ever reaching the turning point of the EKC (see, e.g., Prieur, 2009, Varvarigos, 2014). In addition, some developed economies achieve economic gains accompanied by environmental sacrifice, which contradicts the notion of the downward stretch of the EKC to the right of this turning point (Fact 2).[1] These two facts imply that there are more development paths with different environmental outcomes than what is suggested by the EKC. This paper proposes a framework that can explain these possible additional paths through a behavioral mechanism, and offer some policy recommendations given this scenario.

The workhorse of our analysis is a dynamic general equilibrium model of endogenous growth with a non-constant rate of time preference (endogenous discounting).[2] It is important to provide some motivation for this aspect in terms of how this concept has evolved from a historical perspective alongside economic development. This vital link has been provided in the recent paper by Galor and Ozak (2016), where their theory suggests that in societies where the ancestral population experienced a higher crop yield (for a given crop growth cycle), the rewarding experience in agricultural investment set in motion the forces which have gradually increased the

[1] One example widely cited here is the case of China. We elaborate further on that later in our Introduction.

[2] Following the discounted utility model of Samuelson (1937), where the motives underlying intertemporal choice decisions could be captured by a single parameter, the discount rate, theories such as hyperbolic discounting posit declining discount rates over time (see, for instance, Thaler, 1981); also, the hyperbolic functional form appears to fit the data better than its exponential counterpart. Further, the discounted utility model assumes that the discount rate should be the same for all types of goods and all categories of intertemporal decisions. Certain empirical regularities, such as gains being discounted more than losses, and small amounts being discounted more than large amounts, seem to contradict this assumption (see Frederick et al., 2002, for details). Freeman et al. (2015) contend that an endogenous discount rate is important for the evaluation of long-term projects, when future generations are taken into consideration.

representation of traits for higher long-term orientation among the descendants of individuals who resided in such geographical regions in the contemporary period.[3] That paper is instructive in drawing a parallel with what could be expected in an environmental context: similar to agricultural investment, better nurturing and protection of the environment in a particular era could result in higher long-term orientation among members of subsequent generations; so the rate of time preference could be expected to decrease over time as environmental quality improves (see, among others, Yanase, 2011; Vella et al., 2015; Chu, 2016; Lines, 2005; Pittel, 2002). Along similar lines, Viscusi et al. (2008) contend that behavioral evidence shows that people who value the importance of natural resources have low rates of discount, which is what we consider in our analysis.

In our study, better environmental quality increases the preferences for the future relative to the current period, raises savings and growth and, in turn, the resources to abate pollution activities. On the flip side, there exists a vicious cycle of a high discount rate and low savings, environmental quality and growth. This mechanism of multiple equilibria can help us explain Fact 1, as a phenomenon that is driven by deep behavioral roots. In this context, in contrast to the commonly held view, advancements in productivity may not help countries with poor environmental quality. Interestingly, while, with productivity gains, people become more productive and are able to enjoy higher incomes, they spend a higher proportion of their incomes to consumption because of their low long-term orientation (that increases pollution), rather than savings (that enhance resources for abatement policies). In turn, we argue that an escape from a low-level equilibrium can take place through a behavioral change. To this end, we consider endogenous (Ramsey welfare-maximizing) fiscal policy, which leads to the elimination of the 'bad' equilibrium (low growth and environmental quality). Moreover, we offer an explanation for Fact 2 by showing that under increases in total factor productivity (TFP), which foster growth, some advanced economies may perform badly in environmental terms. This happens

[3]Moreover, their theory further proposes that societies that benefited from the expansion of suitable crops in the post-1500 period experienced further gains in the degree of long-term orientation. Consistent with these predictions, their empirical analysis establishes that geographical variations in the natural return to agricultural investment have had a persistent effect on the distribution of time preference across societies.

because the additional resources generated via higher growth lead to increases in consumption and, in turn, in environmental degradation and impatience, despite the endogenous shift of government expenditure towards the environment.

Regarding environmental dynamics, we follow Jouvet et al. (2005) and Angelopoulos et al. (2013). In particular, the stock of environmental quality, which remains constant in the long-run (Eliasson and Turnovsky, 2004), depends negatively on pollutants and positively on abatement expenditures, undertaken by the government (see Mohtadi, 1996; Smulders and Gradus, 1996; Byrne, 1997; Liddle, 2001; Bretschger and Smulders, 2006; Managi, 2006).[4] Environmental degradation occurs via aggregate consumption. This is a realistic assumption, given that consumption of natural resources and the consumption of energy-intensive luxury goods are important sources of pollution. Consumption of automobile services (particularly when vehicles are without well-functioning catalytic converters) leads to significant air pollution. Household wastes and municipal sewage, when dumped into waterways, lead to widespread water pollution. Consumption of various electronic appliances leads to radiation and sound pollution. These are all by-products of consumption activities. Other examples include the consumption of fossil fuels like coal, wood, kerosene oil, etc., in the rural areas of developing countries.[5] As regards environmental preservation, a distinct feature of our model is that the effectiveness of public abatement expenditure is enhanced by the amount of public infrastructure per unit of output. To this end, one can cite examples of investment in green infrastructure projects, e.g., efficient management of stormwater, climate adaptation and provision of green space, which can supplement the direct benefits from abatement.[6]. See Quaas (2007) for a

[4]The proportion of government expenditure in total expenditure on abatement is relatively high in many countries (see e.g. Hatzipanayotou et al., 2003; Haibara, 2009).

[5]For the consumption-caused pollution hypothesis, see John and Pecchenino (1994), Howarth (1996), Liddle (2001), Egli and Steger (2007), Quaas (2007), Bertinelli et al. (2008), Gupta and Ray Barman (2009). In Bretschger and Smulders (2006), pollution is modelled as a by-product of either capital or consumption, while in Michael et al. (2015) pollution is generated via both production and consumption. Another strand of the literature considers physical capital as the source of pollution: see, e.g., Cassou and Hamilton (2004); Benarroch and Weder (2006).

[6]As an example, consider public spending on transport infrastructure. We know that the conditions of roads are adversely affected by acid rain (potholes and cracks develop periodically). If the government spends to maintain such roads (and even spends on new roads), while at the same time spending on abatement (i.e., spending on research to tackle the causes of acid rain and lessen its harmful effects), then better quality roads will mean less corrosive effect of acid rain on roads, so a unit of spending on abatement will be more efficient

spatial equilibrium model where the harmful consequences of pollution may be reduced by supplying adequate infrastructure. Public spending on human capital that could enable the educated to carry out R&D activities to bolster the environment is yet another example.

Our results for the decentralized economy show that there emerge multiple (two) equilibria: one, a 'bad', low-growth, equilibrium, and the other, a 'good', high-growth, equilibrium. The former typifies the case of a resource-poor country, with consumers having a high degree of impatience and high consumption propensity, which, in addition to low growth also results in high pollution and low environmental quality (see Fact 1). Exactly the opposite holds for the other equilibrium, the prototype of an advanced economy, characterized by a low rate of time preference, low consumption-to-capital ratio, low pollution, good environmental quality and high growth. Our findings can be compared with Schumacher (2009) where multiplicity of steady states occurs since when an agent is very poor, then increases in consumption are necessary for survival, and the agent is so impatient that the preferences are clearly directed towards the current period. By contrast, when overall wealth has already been built up sufficiently, the discount rate becomes relatively low and people plan ahead for the future. We also study the implications of higher TFP for both equilibria. For the 'bad' equilibrium, despite the increase in output, consumption rises more than in proportion, pollution is higher and environmental quality is lower; the resulting higher degree of impatience contributes to even lower growth. This finding suggests that productivity increases may not help such economies to escape the trap involving low growth and environmental quality, and motivates us to study the implications for the pursuit of second-best (Ramsey) policy.[7]

Under Ramsey fiscal policy, we no longer have the existence of two equilibria. The government, by being able to exercise two additional instruments, other than its overall expenditure, is able to channel the economy in a direction whereby it can avoid being trapped in the 'bad' equilibrium, so only the high-growth equilibrium (with good environmental quality) ensures. The implications of a TFP increase for the Ramsey planner in this case could well be to al-

[7]To explain poverty traps and prosperity across countries, without environmental considerations, Shao and Zhang (2018) recently study the multiplicity of balanced growth paths and the role of fiscal policy in avoiding such traps in an extended Lucas model with human capital externalities.

locate a higher proportion of tax revenues towards environmental protection. However, this appears not to be enough to counteract the deterioration of environmental conditions from increased aggregate consumption. As a result, such fast-growing economies obtain economic gains accompanied by environmental sacrifice (see Fact 2).

The contribution of our paper to the literature is, therefore, twofold. First, we extend the literature on growth, environment and endogenous discounting (see, among others, Yanase, 2011; Vella et al., 2015; Chu, 2016; Pittel, 2002). We model environmental resources as stock, which enables us to capture the existence of environmental and economic poverty traps (Fact 1) through the behaviour of agents due to low environmental quality.[8] Also, differently to Vella et al. (2015), here time preference depends solely on environmental quality, which enables us to better examine the link between the two, and the environmental law of motion takes into account the effect of 'green' infrastructure on the efficiency of abatement policies. More broadly, our paper also contributes to the literature on growth and the environment by developing a model in which, unlike Vella et al. (2015), balanced growth is consistent with constant environmental quality, under explicit modelling of environmental dynamics and fiscal policy. Eliasson and Turnovsky (2004) is another important contribution here, without, however, considering public environmental policies.

Second, we provide an explanation for the poor environmental performance of advanced countries (Fact 2) through a normative aspect (endogenous policy). To the best of our knowledge, this is the first paper showing that advanced countries could end up hurting the environment under a second-best policy objective. This result goes some way towards explaining the situation in fast-growing economies like China over the past decade, where a rise in productivity and high growth exist side by side with environmental degradation. Research and development (R&D) expenditure (as % of GDP) in China rose continuously over the years (from 1.32 in 2005 to 1.98 in 2012). With a decadal growth rate of 10% and GDP per capita (in current US$) that rose from 1,731.1 in 2005 to 6,807.4 in 2013, CO_2 emissions went up from 4.4 in 2005 to 6.2 in

[8] In Yanase (2011) and Chu et al. (2016) the decentralized equilibrium is unique.

2010.[9]

The rest of the paper is structured as follows. Section 2 sets up the model. Section 3 studies the properties of the decentralized economy. Section 4 analyzes the role of Ramsey government and the effect of a TFP increase. Finally, section 5 concludes.

2 The model

This section presents the set-up of our closed-economy model. The main features are as follows: (a) households derive utility from consumption and environmental quality, which has a public good character; (b) the subjective discount rate is a negative function of environmental quality, taken as given by the agents; (c) public infrastructure provides production externalities to firms and enhances the efficiency of abatement technology; (d) consumption generates environmental pollution; (e) the government imposes a tax on output and uses the collected tax revenues to finance infrastructure and abatement.

2.1 Households

The economy is made up of a large number of identical, infinitely-lived households, normalized to unity, and each one seeks to maximize the present discounted value of the lifetime utility:

$$\int_0^\infty u(C_t, N_t) \exp\left[-\int_0^t \rho(N_v)dv\right] dt \tag{1}$$

where $u(C,N) = v\ln C + (1-v)\ln N$ is the instantaneous utility function with $0 < v \leq 1$ measuring how much agents value consumption, C, vis-à-vis the stock of economy-wide natural resources, interpreted as an index for environmental quality, N.[10] In turn, $\rho(N)$ denotes the

[9] If we consider R&D researchers (per million people) or patent applications, the former rose from 890 in 2010 to 1020 in 2012 and the latter increased about 2.5 times between 2010 and 2013. We link these trends to the Ramsey policy outcome rather than to a pure EKC effect (where a decline in environmental quality occurs with higher economic growth) because countries like China (and India to some extent) can be viewed as economies that are to the right of the EKC.

[10] We use a logarithmic utility function to avoid unnecessary structure and to focus on the *intertemporal* effect of endogenous discounting. For a detailed analysis on the *intra*temporal concavity of the utility function (with a constant rate of time preference), see Kim (2005).

endogenous rate of time preference (RTP), which is assumed to depend negatively on environmental quality, i.e. $\rho_N \leq 0$.[11] The assumption that a higher level of environmental quality lowers individual impatience follows Yanase (2011), Vella et al. (2015), Chu (2016), Pittel (2002) and is motivated by behavioral evidence showing that people who value the importance of natural resources have low rates of discount (Viscusi et al., 2008).[12] Further, we assume that there exists a lower positive bound for the RTP, denoted by $\breve{\rho}$, i.e. $\lim_{(N)\to 0} \rho(N) = \breve{\rho} > 0$.

Households save in the form of capital and receive dividends, π. The budget constraint of the household is given by:

$$\dot{\zeta} + C = r\zeta + wL + \pi \tag{2}$$

where a dot over a variable denotes a derivative with respect to time, r is the after-tax rental rate on capital, w the after-tax wage rate from inelastic labor, L, ζ denotes financial assets and the initial asset endowment $\zeta(0) > 0$ is given. The household acts competitively by taking prices, policy, and environmental quality as given. The latter is justified by the open-access and public good features of the environment. The control variables are the paths of C and $\dot{\zeta}$, so that the first-order conditions include the constraint (2) and the Euler equation below:

$$\frac{\dot{C}}{C} = r - \rho(N) \tag{3}$$

Notice that environmental quality affects consumption growth positively through the RTP, and thus plays an implicit 'productive' role in the economy.

2.2 Firms

The production function of the single good in this economy is given by:

$$Y = AK^a L^{1-a} {K_g}^{1-a} \tag{4}$$

[11]We retain the equality sign to allow for comparisons with the case of constant RTP.

[12]Intuitively, interpreting agents as dynasties/families, we can conceive that in economies with better environmental quality agents understand better its positive effects on their health and on their children acquiring education. In turn, they care more about future generations, and they sacrifice their consumption (which would have otherwise added to the pollution) for higher environmental quality and consumption of future generations.

where Y denotes output, $0 < a < 1$ denotes the share of physical capital, K, in the production function, K_g refers to the public capital stock (e.g. infrastructure), and A represents TFP.[13] labor endowment is normalized to unity and we assume no population growth. The law of motion for the public capital stock is given by:

$$\dot{K}_g = G - \delta_{K_g} K_g \tag{5}$$

where δ_{K_g} denotes the depreciation rate and G is government investment in public capital. The initial capital stock $K_g(0) > 0$ is given. The firm maximizes profits, π:

$$\pi = (1 - \tau)Y - (r + \delta_K)K - wL \tag{6}$$

where $0 < \tau < 1$ is a tax rate on output, δ_K is the depreciation rate of private capital, and its summation with r forms the rental cost of capital. The firm acts competitively by taking prices and policy instruments as given. The first-order conditions equates the marginal productivity of capital to its rental cost and the marginal product of labour to the wage rate:

$$r + \delta_k = Aa(1 - \tau)L^{1-a} \left(\frac{K}{K_g}\right)^{a-1} \tag{7}$$

$$w = A(1 - a)(1 - \tau)L^{-a} \left(\frac{K}{K_g}\right)^{a} K_g \tag{8}$$

2.3 Motion of environmental quality

Following Jouvet et al. (2005) and Angelopoulos et al. (2013), we assume that the stock of environmental quality evolves over time according to:

$$\dot{N} = (1 - \delta_N)\bar{N} - (1 - \delta_N)N - D \tag{9}$$

[13] This specification follows the strand of endogenous growth theory assuming that the government can invest in productive public capital, which stimulates aggregate productivity (see e.g., Barro, 1990; Futagami, et al., 1993).

where $\bar{N}$ denotes environmental quality without degradation, D, and $\delta_N \in (0,1)$ is the degree of environmental persistence.[14] The initial stock $N(0) > 0$ is given.

Environmental degradation, D, is a positive function of pollution emissions, P, and a negative function of public abatement expenditures, E, i.e. $D_P(P,E) > 0$ and $D_E(P,E) < 0$:

$$D = D(P,E) = \frac{P}{\theta E} \tag{10}$$

where $0 < \theta(K_g, Y) \leq 1$ denotes the efficiency of the abatement technology in alleviating environmental degradation which is an endogenous variable in our model.[15]

We further assume that P occurs as a by-product of consumption:

$$P = sC \tag{11}$$

where $0 < s < 1$ is a parameter that quantifies the detrimental effect of consumption on the environment.[16]

In the same vein as Andreoni and Levinson (2001) and Quaas (2007), we assume that the efficiency of expenditures to abate the environment depend on the size of the economy and the level of infrastructure. In particular, we assume that the efficiency of abatement technology is a positive function of infrastructure stock as a share of total output, $\frac{K_g}{Y}$:

$$\theta \equiv \theta\left(\frac{K_g}{Y}\right) = \xi \frac{K_g}{Y} \tag{12}$$

where $\xi > 0$ is the parameter that captures the effect of infrastructure on the efficiency of abatement. Intuitively, higher investment on public infrastructure complements public expenditures on abatement and makes it possible to clean the environment in a more efficient way. The most

[14] For further details on the inclusion of the parameter $\bar{N}$, see Jouvet et al. (2005, p. 1599).

[15] For papers with publicly financed abatement see e.g. Ligthart and van der Ploeg (1994); Greiner (2005); Pérez and Ruiz (2007); Gupta and Barman (2010); Vella et al. (2015). An interesting extension for future research would be to model both private and public abatement.

[16] As in Andreoni and Levinson (2001), Egli and Steger (2007), Gupta and Ray Barman (2009), Mariani et al. (2010), and Michael et al. (2015), we consider a linear relationship between pollution flows and consumption activities for the sake of simplicity.

obvious example here is the importance of (the lack of adequate) public infrastructure, such as roads, in some developing countries when it comes to assessing the impact of government policies on environmental protection.

Another example is to consider effective stormwater management. This involves providing drainage support in urban areas, as without such efforts, water does not infiltrate the ground due to much of the surface being impervious, and causes pollution and flooding via runoffs.[17] One can also consider environmental infrastructure, referring to infrastructure that provides cities and towns with water supply, waste disposal, and pollution control services, as component of what we label here 'public capital'. The same holds for public spending on education. Publicly funded education would enable people to create more technologies to clean the environment, resulting in a unit spending on abatement to generate even greater benefits. The negative effect of income comes from congestion effects on public investment (see Turnovsky, 2000).[18]

2.4 Government budget constraint

The government spends G on infrastructure and E on environmental policy, and collects revenues through a tax on output, $0 < \tau < 1$. Assuming a balanced budget, we can write:

$$G + E = \tau Y \tag{13}$$

Equivalently, (13) can be written as:

$$G = b\tau Y \tag{14}$$

$$E = (1 - b)\tau Y \tag{15}$$

[17]For example, permeable pavements in parks and parking lots, and wetlands for management of stormwater runoff, have been constructed in the US city of Philadelphia. Also, in Singapore, stormwater runoff is treated without the use of chemicals but naturally, through the use of plants and soil media, so that cleaner water is discharged into waterways and eventually to reservoirs.

[18]While we argue that infrastructure increases the abatement efficiency, it has a negative effect on pollution (through its positive relation with output) and, in turn, on consumption. This general equilibrium effect is captured in our results.

where $0 < b < 1$ is the fraction of tax revenue used to finance infrastructure and $0 < (1-b) < 1$ is the fraction that finances environmental investment. Thus, government policy can be summarized by the two policy instruments, τ and b.

3 Decentralized competitive equilibrium

In this section we solve for a DCE, which holds for any feasible policy, and analyze its properties.

Definition 1 *The DCE of the economy is defined for the exogenous policy instruments τ and b, the factor prices r, w, and the aggregate allocations K, K_g, N, G, E, C such that:*

i) Individuals solve their intertemporal utility maximization problem by choosing c and ζ, given the policy instruments and the factor price.

ii) Firms choose K and L in order to maximize their profits, given the factor prices and aggregate allocations.

iii) All markets clear, which implies for the capital market $\zeta = K$ (assets held by agents equal the private capital stock) and the labour market $L = 1$.

iv) The government budget constraint holds.

Combining (1)-(15) and assuming for the rest of the paper, without loss of generality, that $\delta_K = \delta_{K_g} = \delta$, it is straightforward to show that the DCE is given by:

$$\frac{\dot{C}}{C} = \left[Aa(1-\tau)\left(\frac{K}{K_g}\right)^{a-1} - \delta - \rho(N) \right] \tag{16}$$

$$\frac{\dot{K}}{K} = A(1-\tau)\left(\frac{K}{K_g}\right)^{a-1} - \frac{C}{K} - \delta \tag{17}$$

$$\frac{\dot{K_g}}{K_g} = Ab\tau\left(\frac{K}{K_g}\right)^{a} - \delta \tag{18}$$

$$\dot{N} = (1-\delta_N)\bar{N} - (1-\delta_N)N - \left(\frac{s}{\xi(1-b)\tau}\frac{C}{K_g}\right) \tag{19}$$

Equations (16)-(19) summarize the dynamics of the economy. Owing to the presence of environmental quality in (16), equations (16)-(18) cannot be solved independently of (19).

Finally, the transversality condition for this problem is given by:

$$\lim_{t\to\infty} \frac{K(t)}{C(t)} \exp\left[-\int_0^t \rho\left(N_s\right) ds\right] = 0 \tag{20}$$

The balanced growth path (BGP) in this economy is defined as a state where variables C, K, K_g, Y grow at a constant rate, g, and environmental quality is constant. Following usual practice, we will reduce dimensionality to facilitate analytical tractability by defining the following auxiliary stationary variables, $\omega \equiv \frac{C}{K}$ and $z \equiv \frac{K}{K_g}$. Then, it is straightforward to show that the dynamics of (16)-(19) are equivalent to the dynamics of the following system of equations:

$$\frac{\dot{\omega}}{\omega} = A(a-1)(1-\tau)z^{a-1} - \rho\left(N\right) + \omega \tag{21}$$

$$\frac{\dot{z}}{z} = A(1-\tau)z^{a-1} - Ab\tau z^{a} - \omega \tag{22}$$

$$\dot{N} = (1-\delta_N)\bar{N} - (1-\delta_N)N - \left(\frac{s}{\xi(1-b)\tau}\omega z\right) \tag{23}$$

It follows that at the BGP $\frac{\dot{\omega}}{\omega} = \frac{\dot{z}}{z} = \frac{\dot{N}}{N} = 0$. Hence, in our model balanced growth is consistent with constant environmental quality, under explicit modelling of environmental dynamics and fiscal policy. As emphasized in Eliasson and Turnovsky (2004), this important feature was rarely present in existing growth models with environmental resources. Then (22) implies that the long-run ratio of consumption to private capital, $\hat{\omega}$, is expressed as a function of the long-run ratio of private capital to public capital, $\hat{z}$, by:

$$\hat{\omega}(\hat{z}) = A(1-\tau)\hat{z}^{a-1} - Ab\tau\hat{z}^{a} \tag{24}$$

and (23) implies that the long-run value for environmental quality is given by:

$$\hat{N}(\hat{z}) = \bar{N} - \Xi[(1-\tau)A\hat{z}^{a} - b\tau A\hat{z}^{a+1}] \tag{25}$$

where $\Xi(b,\tau;\delta_N,s) \equiv \frac{s}{(1-\delta_N)\xi(1-b)\tau}$.

Substituting then (24)-(25) in (21), given that $\frac{\dot{N}}{N}=0$, we get that $\hat{z}$ is determined by:

$$\Phi(\hat{z}) \equiv -Ab\tau\hat{z}^a + Aa(1-\tau)\hat{z}^{a-1} - \rho(\bar{N} - \Xi[(1-\tau)A\hat{z}^a - b\tau A\hat{z}^{a+1}]) = 0 \qquad (26)$$

Providing there exists a solution $\hat{z} > 0$ in (26), the balanced growth rate, g, is then determined by (18). Assuming equilibrium existence, equations (24)-(26) imply the following:

Proposition 1 *Endogenous efficiency of abatement technology in public capital and endogenous subjective discount rate in environmental quality can lead to multiple long-run growth equilibria. In particular,*

Case 1 (Uniqueness): if $Aa(1-\tau)(\frac{a(1-\tau)}{b\tau})^{a-1} - Ab\tau(\frac{a(1-\tau)}{b\tau})^a < \check{\rho}$, *then the equilibrium is unique.*

Case 2 (Multiplicity): if $Aa(1-\tau)(\frac{a(1-\tau)}{b\tau})^{a-1} - Ab\tau(\frac{a(1-\tau)}{b\tau})^a > \check{\rho}$, *then there can be two equilibria, associated with different growth rates ranked* $g_1 < g_2$ *where* $\hat{\rho}_1 > \hat{\rho}_2$, $\hat{\omega}_1 > \hat{\omega}_2$, $\hat{z}_1 < \hat{z}_2$, $\hat{N}_1 < \hat{N}_2$.

Proof. See Appendix A. ▪

Corollary 1 *There exist parameter values such that both equilibria of Case 2 in Proposition 1 are saddle-path stable.*

Proof. *See Appendix A* ▪

Proposition 1 states that endogenous time preference to environmental quality and endogenous efficiency of public abatement to infrastructure spending can lead to multiple solutions for $\hat{z}$, and, in turn, multiple equilibria in the DCE. Hence, although the instantaneous utility and production technology functions satisfy the standard concavity assumptions, the existence of a unique positive balanced growth rate is not guaranteed here. Also, according to Corollary 1, there exist parameter values where both equilibria are stable and, in turn, both are meaningful.

In particular, our model solves for two long-run equilibria: a low-(high-) growth one with low (high) environmental quality, a high (low) rate of time preference, a high (low) consumption-capital ratio and a low (high) physical-to-public capital ratio. This reflects the characteristics of a 'self-defeating' ('self-fulfilling') equilibrium that results from our model set-up. In the former case, a typically poor economy, the propensity to consume is larger, and this generates more pollution (a by-product of c) and a lower environmental quality. This, and the higher consumption propensity, ties in with a high value for the degree of impatience, a higher $\hat{\omega}$ ratio, and a lower $\hat{z}$ ratio. The higher degree of impatience in turn leads to a lower growth rate, which is self-defeating in the sense that a kind of vicious cycle of lower environmental quality and low growth propagates to keep the economy in a 'low-level equilibrium trap' situation**,** corresponding to Fact 1 in the Introduction**.** On the other hand, in the other type of economy (which is rich), lower consumption propensity, better environmental quality and lower impatience jointly deliver a high-growth outcome, which is self-sustaining. Example 1 provides a numerical illustration of our analytical result.[19]

Example 1 *Assume a specific time preference function that satisfies our assumptions,* $\rho(N) = -\gamma \times (N) + \breve{\rho}$*,* $\gamma > 0$ *and parameter values of the growth literature,* $\alpha = 0.5$*,* $A = 0.65$*,* $\delta = 0.14$*,* $\delta_N = 0.9$*,* $\xi = 0.4$*,* $s = 1$*,* $\bar{N} = 20$*,* $\tau = 0.561$*,* $b = 0.751$*,* $\breve{\rho} = 0.2$*, and* $\gamma = 1$*. Then, there exist two equilibria, a 'low-growth' equilibrium with relatively lower growth* $g_1 = 0.010$*, lower environmental quality* $\hat{N}_1 = 0.089$*, and higher rate of time preference* $\hat{\rho}_1 = 0.111$*, and a 'high-growth equilibrium' with higher growth* $g_2 = 0.035$*, better environmental quality* $\hat{N}_2 = 0.151$ *and lower rate of time preference* $\hat{\rho}_2 = 0.048$*.*

The values of the economic parameters in this example are as in most dynamic general equilibrium calibration and estimation studies. Specifically, the value used for the productivity of private capital in the production function, α, comes from Economides and Philippopoulos (2008) and Dioikitopoulos and Kalyvitis (2010). Following common practice, we use the TFP,

[19]In all numerical results presented in the paper, we have rounded off to three (or sometimes four) decimal digits. We have also verified that both equilibria are saddle-path stable. Stability properties of both equilibria presented in Example 1 are examined in Appendix A.

A, as a scale parameter to help us get numerical values for the endogenous variables that can help the reader visualize the qualitative behavior of the model, as implied by our analytical results. We set the value for the detrimental effect of consumption on the environment, s, following Economides and Philippopoulos (2008). For computational tractability, we employ a linear time preference function, $\rho(N) = -\gamma \times (N) + \breve{\rho}$, $\gamma > 0$, (see, e.g., Pittel, 2002; Dioikitopoulos and Kalyvitis, 2010), which is rich enough to obtain our main results. The chosen values for the lower bound, $\breve{\rho}$, and slope, γ, help us calibrate values for ρ in line with the literature. In particular, the highest RTP values reported for the low-growth regime are close to that in Elbasha and Roe (1996), while those reported for the high-growth regime are in the range commonly employed in the growth literature.

3.1 Productivity, growth and environmental quality

In this sub-section, we study the effect of a change in productivity, as given by an increase in TFP, on environmental quality and growth.

Proposition 2 *If the response of time preference to environmental quality is relatively high, then for the low growth, bad environment equilibrium* (g_1, $\hat{N}_1$)*, an increase in productivity,* A*, has a negative effect on steady-state environmental quality,* $\frac{\partial \hat{N}_1}{\partial A} < 0$*, the long-run economic growth rate,* $\frac{\partial g_1}{\partial A} < 0$*, and the physical to public capital ratio,* $\frac{\partial \hat{z}_1}{\partial A} < 0$*, while it has a positive effect on the consumption to physical capital ratio,* $\frac{\partial \hat{\omega}_1}{\partial A} > 0$.

Proof. *Appendix B.* ▪

Proposition 2 states that for economies trapped in an equilibrium of low environmental quality, high impatience and low growth (Case 2 of Proposition 1), an increase in productivity will further deteriorate environmental quality and long-run growth. This is an analytical result that comes in contrast with common wisdom in cases of multiple equilibria, where solely productivity changes may help an economy escape from an equilibrium with low environmental quality and output growth. Intuitively, if the environmental quality is low, individuals' long-term orientation is weak (i.e., they are short-sighted in their outlook) and, in turn, their prospensity to save is low. Then, while with an increase in productivity, their income initially increases (first order effect), agents increase their consumption proportionally more than their savings, $\frac{\partial \omega_1}{\partial A} > 0$. Higher consumption increases pollution and lowers savings, leading to lower income and growth in the future. Subsequently, lower growth results in a lower tax base (for a given tax rate), and the resources for abatement become insufficient to restore the environmental damage (second order effect). If patience is strongly related with environment (negatively), then the first order, positive, effect of productivity on income (static) is outweighted by the second order, dynamic, effect of increase in consumption and, in turn, pollution. This results in lower environmental quality and growth as Proposition 2 formally addresses. Taking into account Corollary 1, there exist parameter values that such an equilibrium is stable, and, in turn, a positive productivity shock results to a stable equilibrium with lower growth.

For the equilibrium with high environmental quality and growth, an increase in productivity can have a positive effect on environmental quality as agents direct resources proportionally more to capital formation rather than consumption. However, the final outcome depends on how the government spends the additional resources towards abatement vis-a-vis infrastructure.[20] Given the complexity of the system in this case, we resort to a numerical example below, using the same parameterization as in the previous example, except for the TFP parameter which we increase from $A = 0.65$ to $A = 0.66$.

Example 2 *Assume a specific time preference function that satisfies our assumptions,* $\rho(N) = -\gamma \times (N) + \breve{\rho}$, $\gamma > 0$ *and parameter values of the growth literature,* $\alpha = 0.5$, $A = 0.66$, $\delta = 0.14$, $\delta_N = 0.9$, $\xi = 0.4$, $s = 1$, $\bar{N} = 20$, $\tau = 0.561$, $b = 0.751$, $\breve{\rho} = 0.2$, *and* $\gamma = 1$. *Then, there exist two equilibria, a 'low-growth' equilibrium with relatively lower growth* $g_1 = 0.004$, *lower environmental quality* $\hat{N}_1 = 0.064$, *and higher rate of time preference* $\hat{\rho}_1 = 0.136$, *and a 'high-growth equilibrium' with higher growth* $g_2 = 0.045$, *better environmental quality* $\hat{N}_2 = 0.168$ *and lower rate of time preference* $\hat{\rho}_2 = 0.032$.

A higher value of A, representing higher productivity, typically raises output. For an econ-

[20]We analyze this point thoroughly in the next section.

omy with high-growth and good environmental quality, such a rise in TFP leads to even higher growth (from $g_2 = 0.035$ in Example 1 to $g_2 = 0.045$ in Example 2), better environmental quality (from $\hat{N}_2 = 0.151$ in Example 1 to $\hat{N}_2 = 0.168$ in Example 2) and lower degree of impatience (from $\hat{\rho}_2 = 0.048$ in Example 1 to $\hat{\rho}_2 = 0.032$ in Example 2). However, for an economy characterized by a low level of environmental quality, and in turn less patience and lower propensity to save, an increase in output driven by higher TFP raises consumption proportionately more than private saving and causes more pollution. A slowly growing economy will also end up with a lower tax base, and will have lower resources to allocate to both infrastructure and abatement, so that the environmental quality deteriorates further, raising the degree of impatience and giving rise to a further growth-reducing effect. This analytical result of Proposition 2 suggests that productivity increases may not help such economies to escape the environmental and economic poverty trap.

4 Ramsey fiscal policy

In this section we endogenize fiscal policy, as summarized by the time paths of the two policy instruments, $0 < \tau < 1$ and $0 < b \leq 1$, by solving for the Ramsey second-best problem of the government. Given a welfare criterion that the government uses to evaluate different allocations, the Ramsey problem for the government is to pick the fiscal policy that generates the competitive equilibrium allocation with the highest value of this criterion.[21]

Definition 2 *A Ramsey Allocation is given under Definition 1 when (i) the government chooses the tax rate, τ, and the allocation of revenues to infrastructure and public abatement, b, in order to maximize the welfare of the economy by taking into account the aggregate optimality conditions of the competitive equilibrium, and (ii) the government budget constraints and the feasibility and technological conditions are met.*

In particular, the government seeks to maximize welfare in the economy subject to the

[21]For second-best policies in models with environmental resources, see e.g Antoniou et al. (2012) and Economides and Philippopoulos (2008).

outcome of the decentralized equilibrium, summarized by (16)-(19). Due to the variable RTP, Pontryagin's maximum principle cannot be applied directly. To solve the problem within the standard optimal control framework, we follow the procedure employed by Obstfeld (1990) and introduce an additional 'artificial' variable that accounts for the development of the accumulated discount rate, $\Delta(t) \equiv \int_0^t \rho(N_v)dv$. Then, the objective of the government is to maximize intertemporal utility:

$$\max U^R = \int_0^\infty (v \ln C + (1-v)\ln N) \exp\left[-\int_0^t \rho(N)dv\right] dt$$

constrained by the competitive equilibrium, (16)-(19), and the derivative of $\Delta(t)$ with respect to time, $\dot{\Delta} = \rho(\cdot)$.

The Hamiltonian of the problem is given by:

$$\begin{aligned}\Lambda^{RSB} &= [\nu \log C + (1-\nu)\log(N)]e^{-\Delta} + \lambda_C C[a(1-\tau)A\left(\frac{K}{K_g}\right)^{a-1} - \delta - \rho(N)] \\ &\quad +\lambda_K\left[(1-\tau)AK^aK_g^{1-a} - C - \delta K\right] + \lambda_{K_g}\left(b\tau AK^aK_g^{1-a} - \delta K_g\right) \\ &\quad +\lambda_N\left[(1-\delta_N)\bar{N} - (1-\delta_N)N - \frac{\psi}{\xi(1-b)\tau}\frac{C}{K_g}\right] + \lambda_\Delta \rho(N)\end{aligned}$$

The first-order conditions of the Ramsey problem include the Euler equation, the growth rates of private capital, public capital and environmental quality, the resource constraint, and the optimality conditions with respect to C, K_g, K, N, τ, b, Δ:

$$\frac{\nu}{C}e^{-\Delta} + \lambda_C\left[Aa(1-\tau)\left(\frac{K}{K_g}\right)^{a-1} - \delta - \rho(N)\right] - \lambda_K - \lambda_N\frac{\psi}{\xi(1-b)\tau}\frac{1}{K_g} = -\dot{\lambda}_C \tag{27}$$

$$\lambda_C C\left[Aa(1-\tau)(a-1)K^{a-2}K_g^{1-\alpha}\right] + \lambda_K\left[A(1-\tau)\alpha\left(\frac{K}{K_g}\right)^{a-1} - \delta\right] + \lambda_{K_g}Ab\tau\alpha\left(\frac{K}{K_g}\right)^{a-1} = -\dot{\lambda}_K \tag{28}$$

$$\lambda_C C \left[Aa(1-\tau)(1-\alpha)K^{a-1}K_g^{-\alpha}\right] + \lambda_K(1-\tau)(1-\alpha)A\left(\frac{K}{K_g}\right)^a + \lambda_{K_g}\left[b\tau(1-\alpha)A\left(\frac{K}{K_g}\right)^a - \delta\right] + \lambda_N \frac{\psi}{\xi(1-b)\tau}\frac{C}{K_g^2} = -\dot{\lambda}_{K_g} \quad (29)$$

$$\frac{(1-\nu)}{N}e^{-\Delta} - \lambda_C C\rho'(N) - \lambda_N(1-\delta_N) + \lambda_\Delta \rho'(N) = -\dot{\lambda}_N \quad (30)$$

$$\lambda_C C\left[-aA\left(\frac{K}{K_g}\right)^{a-1}\right] - \lambda_K A K^a K_g^{1-a} + \lambda_{K_g} AbK^a K_g^{1-a} + \lambda_N \frac{\psi}{\xi(1-b)\tau^2}\frac{C}{K_g} = 0 \quad (31)$$

$$\lambda_{K_g}\tau A K^a K_g^{1-a} - \lambda_N \frac{\psi}{\xi(1-b)^2\tau}\frac{C}{K_g} = 0 \quad (32)$$

$$[\nu \log C + (1-\nu)\log N]e^{-\Delta} = \dot{\lambda}_\Delta \quad (33)$$

where $\lambda_C, \lambda_K, \lambda_{K_g}, \lambda_N, \lambda_\Delta$ are the dynamic multipliers associated with (16)-(19) and $\dot{\Delta} = \rho(\cdot)$. Then, condition $\dot{\Delta} = \rho(\cdot)$ and equations (27)-(33) characterize the dynamics of the Ramsey problem. To obtain the long-run solution of the problem we define the stationary variables: $\omega \equiv \frac{C}{K}$, $z \equiv \frac{K}{Kg}$, $c \equiv \tilde{\lambda}_C C$, $\kappa \equiv \tilde{\lambda}_K K$, $\kappa_g \equiv \tilde{\lambda}_{K_g} K_g$ where $\tilde{\lambda}_i = e^\Delta \lambda_i$. After some algebra the long-run Ramsey equilibrium is given by:

$$\nu + \tilde{c}\left[Aa(1-\tilde{\tau})\tilde{z}^{a-1} - \delta - \rho\left(\tilde{N}\right)\right] - \tilde{\kappa}\tilde{\omega} - \tilde{\lambda}_N \frac{\psi}{\xi(1-\tilde{b})\tilde{\tau}}\tilde{\omega}\tilde{z} = (A\tilde{b}\tilde{\tau}\tilde{z}^a - \delta)\tilde{c} + \tilde{\rho}(\tilde{N})\tilde{c} \quad (34)$$

$$\tilde{\rho}(\tilde{N}) = -\gamma\tilde{N} + \breve{\rho} \quad (35)$$

$$\tilde{c}a(1-\tilde{\tau})(a-1)A\tilde{z}^{a-1} + \tilde{\kappa}\left[(1-\tilde{\tau})A\alpha\tilde{z}^{a-1} - \delta\right] + \tilde{\kappa}_g A\tilde{b}\tilde{\tau}\alpha\tilde{z}^a = (A\tilde{b}\tilde{\tau}\tilde{z}^a - \delta)\tilde{\kappa} + \tilde{\rho}(\tilde{N})\tilde{\kappa} \quad (36)$$

$$\tilde{c}\left[Aa(1-\tilde{\tau})(1-\alpha)\tilde{z}^{a-1}\right] + \tilde{\kappa}A(1-\tilde{\tau})(1-\alpha)\tilde{z}^{a-1} + \tilde{\kappa}_g\left[A\tilde{b}\tilde{\tau}(1-\alpha)\tilde{z}^a - \delta\right] + \tilde{\lambda}_N \frac{\psi}{\xi(1-\tilde{b})\tilde{\tau}}\tilde{\omega}\tilde{z} = (A\tilde{b}\tilde{\tau}\tilde{z}^a - \delta)\tilde{\kappa}_g + \tilde{\rho}(\tilde{N})\tilde{\kappa}_g \quad (37)$$

$$\tilde{c}\left[-aA\tilde{z}^{a-1}\right] - \tilde{\kappa}A\tilde{z}^{a-1} + \tilde{\kappa}_g A\tilde{b}\tilde{z}^a + \tilde{\lambda}_N \frac{\psi\tilde{\omega}\tilde{z}}{\xi(1-\tilde{b})\tilde{\tau}^2} = 0 \quad (38)$$

$$(1-\nu) - \tilde{c}\tilde{\rho}'(\tilde{N})\tilde{N} - \tilde{\lambda}_N \tilde{N}(1-\delta_N) - \tilde{\rho}(\tilde{N})\lambda_N \tilde{N} = 0 \tag{39}$$

$$A\tilde{\kappa}_g \tilde{\tau}\tilde{z}^a - \tilde{\lambda}_N \frac{\psi\tilde{\omega}\tilde{z}}{\xi(1-\tilde{b})^2\tilde{\tau}} = 0 \tag{40}$$

$$A(a-1)(1-\tilde{\tau})\tilde{z}^{a-1} - \tilde{\rho}\,(N) + \tilde{\omega} \tag{41}$$

$$\tilde{\omega} = A(1-\tilde{\tau})\tilde{z}^{a-1} - A\tilde{b}\tilde{\tau}\tilde{z}^a \tag{42}$$

$$\tilde{N} = \bar{N} - \Xi[(1-\tilde{\tau})A\tilde{z}^a - \tilde{b}\tilde{\tau}A\tilde{z}^{a+1}] \tag{43}$$

Equations (34)-(43) describe a system of 10 equations with 10 unknowns, $\tilde{z}$, $\tilde{b}$, $\tilde{\tau}$, $\tilde{\omega}$, $\tilde{N}$, $\tilde{\rho}$, $\tilde{\kappa}_g$, $\tilde{\lambda}_N$, $\tilde{\kappa}$, $\tilde{c}$. Due to the complexity of the system, we resort to numerical simulations. We aim, first, to examine the role of Ramsey government in equilibrium selection, and, second, to examine the effect of an increase in productivity, A, on the patience level, policy instruments, growth and the environment. We follow exactly the same numerical parameter values and time preference specification as in the DCE analysis, except for the endogenous policy instruments, $\tilde{\tau}$ and $\tilde{b}$, that are derived endogenously through the Ramsey objective. In addition, we set the environmental preference parameter vis-a-vis consumption, which does not affect the DCE dynamics, equal to 0.5, i.e. $v = 0.5$.[22]

Out of the two equilibria (that existed under the DCE), the 'bad' equilibrium is eliminated under optimal fiscal policy to yield a unique equilibrium with high growth and environmental quality. In particular, the government has sufficient policy instruments to a) select the equilibrium and b) internalize the externality to maximize welfare. Selection is feasible since intertemporal complementarities, which fuel multiplicity and are external to the agents in the DCE, are internalized under Ramsey taxation. Selection of equilibrium occurs because the government chooses the aggregate endowments (such as the stock of environment and infrastructure which are taken as given in the DCE) to select a consumption and saving path that puts the economy in the high environmental quality and growth equilibrium and, then, uses the policy instruments to attain the welfare-maximizing objective.

[22]Sensitivity analysis, which has been performed with a large range of parameter values, is available upon request.

For example, for $A = 0.65$, we can see in the first row of Table 1 that the Ramsey problem provides a unique equilibrium. If we use the values for the endogenous tax-spending instruments from the Ramsey problem ($\tau = \tilde{\tau} = 0.56195$, $b = \tilde{b} = 0.75099$) in (24)-(26), while using (18) for the growth rate, the DCE solution gives two equilibria (a 'low-growth' equilibrium with relatively lower growth $g_1 = 0.015$, lower environmental quality $\hat{N}_1 = 0.102$, and higher rate of time preference $\hat{\rho}_1 = 0.098$, and a 'high-growth equilibrium' with higher growth $g_2 = 0.030$, better environmental quality $\hat{N}_2 = 0.140$ and lower rate of time preference $\hat{\rho}_2 = 0.060$).

Regarding changes in productivity, our numerical results in Table 1 show that higher TFP leads to higher output growth and lower environmental quality in the long run. We can see that public capital per unit of private capital, $\tilde{z}$, rises, acting as the engine of growth. At the same time, the utilitarian government has incentives to generate higher tax revenues by choosing a higher tax rate and to reallocate public revenues from infrastructure to the environment. Notably, in response to a TFP increase, the Ramsey government allocates a higher share of revenues to the environment vis-a-vis infrastructure, despite the positive impact of the latter on environmental dynamics through the endogenous efficiency of public abatement. In other words, even when public infrastructure exerts a positive effect on both growth and the environment, the optimal choice of the government is to reallocate resources towards the environment, and not infrastructure, in the face of positive technological developments in the economy. This results from the strong effect of environmental quality on time preference.

However, this policy response is not enough to undo the environmental damage. This outcome is in contrast to the dynamics of the DCE scenario where policy instruments are exogenous, and helps to explain why some fast-growing economies obtain economic gains accompanied by environmental sacrifice (see Fact 2 in the Introduction).

Table 1. Productivity, Ramsey Fiscal Policy, Growth, and the Environment

A	$\tilde{z}$	$\tilde{b}$	$\tilde{\tau}$	$\tilde{\omega}$	$\tilde{N}$	$\tilde{\rho}$	$\tilde{g}$
0.60	0.396	0.7516	0.549	0.273	0.1413	0.0587	0.016
0.61	0.394	0.7515	0.552	0.276	0.1411	0.0589	0.019
0.62	0.391	0.7514	0.554	0.280	0.1409	0.0591	0.022
0.63	0.389	0.7512	0.557	0.283	0.1407	0.0592	0.024
0.64	0.387	0.7511	0.559	0.286	0.1406	0.0594	0.027
0.65	0.384	0.7510	0.562	0.289	0.1404	0.0596	0.030
0.66	0.382	0.7509	0.564	0.292	0.1402	0.0598	0.033

$v = 0.5$, $\alpha = 0.5$, $\delta = 0.14$, $\delta_N = 0.9$, $\xi = 0.4$, $s = 1$, $\bar{N} = 20$, $\gamma = 1$, $\breve{\rho} = 0.2$

Intuitively, after an increase in TFP, the government has an incentive to increase the tax rate to finance expenditures for the environment. However, the consumption effect of higher growth outweighs that of higher abatement expenditures and causes a net increase in pollution and in the rate of time preference (impatience).[23]

Result. *The Ramsey government leads the economy to a unique equilibrium that corresponds to the high-growth DCE regime. Under an increase in productivity, the growth rate increases and the environmental quality deteriorates, despite the endogenous reallocation of public revenues from infrastructure to the environment.*

5 Conclusion

The objective of this paper was to explore possible explanations about two stylized facts: (i) developing countries often stagnate in environmental and economic poverty traps; (ii) some developed economies achieve economic gains accompanied by significant environmental sacrifice, which contradicts the downward-sloping part of the EKC. We have studied an endogenous growth model of the environment, both for a decentralized economy and for an economy with an optimizing (Ramsey) government. In our set-up, the representative agent derived utility from private consumption and environmental quality, and an important feature of the utility function

[23] We have verified that the positive net utility benefit of raising consumption is robust to higher weights on environmental preferences vis-a-vis consumption. The concept of environmental awareness is extensively analyzed in Iosifidi (2016).

was that the rate of time preference was a decreasing function of the quality of the environment. Also, the environment was degraded by pollution, which was a by-product of consumption, and was replenished by abatement activities undertaken by the government. Other than the environment, the government also spent on infrastructure from the income taxes it generated, while balancing its budget. Externalities in production were generated by public capital, and the services from infrastructure positively affected the efficiency of abatement.

Our results for a decentralized economy highlight the existence of multiple equilibria. Here, the 'good' equilibrium is characterized by a higher growth rate driven by a higher private-to-public capital ratio, higher marginal productivity of capital, lower propensity to consume resulting in lower pollution, and higher environmental quality. This, in turn, implies a lower degree of impatience, thus strengthening the growth channel and leading to ever-increasing growth and better environmental quality at the same time. The 'bad' equilibrium is characterized by exactly the opposite effects, pushing the economy in a downward spiral. TFP increases in such an economy have adverse effects on growth and the environment, in sharp contrast to the favourable effects of the same for the 'good' equilibrium. Our model thus offers new insights about developing countries often stuck in environmental and economic poverty traps.

For the case of a Ramsey-government that chooses its policy instruments to maximize agents' welfare, our results demonstrate that the 'bad' equilibrium is eliminated, and the unique equilibrium that remains is a high-growth equilibrium. A productivity increase in this case leads to higher growth and lower environmental quality. The response to higher TFP in this case is for the government to spend a higher proportion of its revenue on the environment. However, this is not enough to offset the polluting effect of higher consumption following a TFP rise. Our model therefore also offers new insights about fast-growing countries obtaining economic gains through environmental sacrifice.

We hope that our findings, by shedding new light into the complex relationships that exist among governmental policy, economic growth and the environment, will be useful for policy-makers focusing on 'green' growth policies.

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Appendix A: Proof of Proposition 1 and Corollary 1

The method will be to separate function $\Phi(\hat{z}) \equiv -Ab\tau\hat{z}^a + Aa(1-\tau)\hat{z}^{a-1} - \rho(\bar{N} - \Xi[(1-\tau)A\hat{z}^a - b\tau A\hat{z}^{a+1}])$ in two functions (Γ and Λ) and find their intersection to solve it. First, let us find the domain of z where there cannot be a fixed point and also the one that there is a well-defined equilibrium ($z > 0$ and $\omega > 0$). We know that $\rho(\bar{N} - \Xi[(1-\tau)A\hat{z}^a - b\tau A\hat{z}^{a+1}]) > 0$. Then an equilibrium exists only if $-Ab\tau\hat{z}^a + Aa(1-\tau)\hat{z}^{a-1} > 0$ for some z, that is $\hat{z} < \frac{a(1-\tau)}{b\tau}$. In other words, for $\hat{z} > \frac{a(1-\tau)}{b\tau}$ then $\Phi(\hat{z}) < 0$ and there is no z that solves $\Phi(\hat{z}) = 0$. Thus, the upper bound of z, z_{ub}, is given by $z_{ub} \equiv \frac{a(1-\tau)}{b\tau}$. So, we restrict our analysis only to the feasible set of equilibria, that is $\hat{z}$: $0 < \hat{z} < \frac{a(1-\tau)}{b\tau}$. Also, since $\frac{a(1-\tau)}{b\tau} < \frac{1-\tau}{b\tau}$ because $a < 1$ then for any $z < \frac{a(1-\tau)}{b\tau}$ is follows that $\hat{\omega} > 0$.

Then, from $\Phi(\hat{z})$ let us define $\Gamma(\hat{z}) \equiv Aa(1-\tau)\hat{z}^{a-1} - Ab\tau\hat{z}^a$ and $\Lambda(\hat{z}) \equiv \rho(\bar{N} - \Xi[(1-\tau)A\hat{z}^a - b\tau A\hat{z}^{a+1}])$. Both $\Gamma(\hat{z})$ and $\Lambda(\hat{z})$ are continuous in $\hat{z}$.

Equation $\Gamma(\hat{z})$ has the following properties:

1. $\lim_{\hat{z}\to 0}\Gamma(\hat{z}) = +\infty$, $\lim_{\hat{z}\to\frac{a(1-\tau)}{b\tau}}\Gamma(\hat{z}) = Aa(1-\tau)(\frac{a(1-\tau)}{b\tau})^{a-1} - Ab\tau(\frac{a(1-\tau)}{b\tau})^a$.
2. $\frac{\partial\Gamma(\hat{z})}{\partial\hat{z}} < 0$, $\frac{\partial^2\Gamma(\hat{z})}{\partial\hat{z}^2} > 0$.

From the properties of $\Gamma(\hat{z})$ is follows that it is a strictly decreasing and convex function in its domain, starts from $+\infty$ and ends at $Aa(1-\tau)(\frac{a(1-\tau)}{b\tau})^{a-1} - Ab\tau(\frac{a(1-\tau)}{b\tau})^a$.

Equation $\Lambda(\hat{z})$ has the following properties:

1. $\lim_{\hat{z}\to 0}\Lambda(\hat{z}) = \rho(\bar{N}) = \check{\rho}$, $\lim_{\hat{z}\to\frac{1-\tau}{b\tau}}\Lambda(\hat{z}) = \rho(0) = \breve{\rho}$.
2. $\frac{\partial\Lambda(\hat{z})}{\partial\hat{z}} = -\rho'(.)\,[aA(1-\tau)\hat{z}^{a-1} - Ab\tau(1+a)\hat{z}^a]$. We have $\frac{\partial\Lambda(\hat{z})}{\partial\hat{z}} > 0$ for $a(1-\tau)\hat{z}^{a-1} - b(1+a)\tau\hat{z}^a > 0 \implies \hat{z} < \frac{a(1-\tau)}{b(1+a)\tau}$ and $\frac{\partial\Lambda(\hat{z})}{\partial\hat{z}} < 0$ for $\hat{z} > \frac{a(1-\tau)}{b(1+a)\tau}$. Thus, $\Lambda(\hat{z})$ has a maximum at $z_{\max} = \frac{a(1-\tau)}{b(1+a)\tau}$. Note also that $z_{\max} < z_{ub}$ as $a > 0$. Then, from the properties of $\Lambda(\hat{z})$ it follows that it is an inverse U-shaped curve starting from $\check{\rho}$ and ending at $\breve{\rho}$.

Assuming equilibrium existence, from the properties of $\Lambda(\hat{z})$ and $\Gamma(\hat{z})$ it follows that there exist one or two positive balanced growth rates. For low values of $\hat{z}$, since $+\infty > \check{\rho}$ we get that $\Gamma(\hat{z})$ lies above $\Lambda(\hat{z})$. Also, for the upper bound value of $\hat{z}$, $\Gamma(\hat{z}) = Aa(1-\tau)(\frac{a(1-\tau)}{b\tau})^{a-1} - Ab\tau(\frac{a(1-\tau)}{b\tau})^a$ and $\Lambda(\hat{z}) = \breve{\rho}$. Since both functions are continuous, if $Aa(1-\tau)(\frac{a(1-\tau)}{b\tau})^{a-1} -$

$Ab\tau(\frac{a(1-\tau)}{b\tau})^{a} < \check{\rho}$, which means that $\Gamma(\hat{z})$ starts above and ends below $\Lambda(\hat{z})$ implying that $\Gamma(\hat{z})$ will cross $\Lambda(\hat{z})$ once and there will exist a unique balanced growth rate. Thus, $Aa(1-\tau)(\frac{a(1-\tau)}{b\tau})^{a-1} - Ab\tau(\frac{a(1-\tau)}{b\tau})^{a} < \check{\rho}$ is a sufficient parametric condition for a unique balanced growth rate.

If $Aa(1-\tau)(\frac{a(1-\tau)}{b\tau})^{a-1} - Ab\tau(\frac{a(1-\tau)}{b\tau})^{a} \geq \check{\rho}$, then there can exist two balanced growth rates because $\Lambda(\hat{z})$ is an inverse U-shaped curve, while $\Gamma(\hat{z})$ strictly monotone and decreasing, so $\Gamma(\hat{z})$ can cross $\Lambda(\hat{z})$ at most two times. Thus, $Aa(1-\tau)(\frac{a(1-\tau)}{b\tau})^{a-1} - Ab\tau(\frac{a(1-\tau)}{b\tau})^{a} \geq \check{\rho}$ is a necessary parametric condition for multiplicity. A graphical illustration is provided in Figure A1.

Ranking of Equilibria: In the case of multiple balanced growth rates, $\Lambda(\hat{z})$ and $\Gamma(\hat{z})$ intersect twice, for $\hat{z}_1$ and $\hat{z}_2$. Let those two balanced growth rates ranked as $\hat{z}_1 < \hat{z}_2$. To find the corresponding ranking of $\hat{\omega}_1$ and $\hat{\omega}_2$ we solve $\hat{\omega}$ in the steady-state, and we take the derivative with respect to $\hat{z}$, $\frac{\partial \hat{\omega}}{\partial \hat{z}} = (a-1)(1-\tau)A\hat{z}^{a-2} - abA\tau\hat{z}^{a-1} < 0$. Thus, $\hat{\omega}$ is a strictly decreasing function of $\hat{z}$, so $\hat{z}_1 < \hat{z}_2 \implies \hat{\omega}_1 > \hat{\omega}_2$. To find the ranking of g_1 and g_2 we take the derivative of g with respect to $\hat{z}$, $\frac{\partial g}{\partial \hat{z}} = b\tau a\hat{z}^{a-1} > 0$. Thus, g is an increasing function of $\hat{z}$, so $\hat{z}_1 < \hat{z}_2 \implies g_1 < g_2$. The ranking for the rate of time preference, $\rho(\hat{z} \cdot \hat{\omega}(\hat{z})) = \Lambda(\hat{z})$, which is a non-monotonic function of $\hat{z}$, comes from the analysis above. As $\Gamma(\hat{z})$ lies above $\Lambda(\hat{z})$ and is monotonically decreasing, it cannot cross twice $\Lambda(\hat{z})$ in its increasing part. Then, $\hat{z}_1 < \hat{z}_2 \implies \Lambda(\hat{z}_1) > \Lambda(\hat{z}_2) \implies \rho_1 > \rho_2$. Also for $\hat{z}_1 < \hat{z}_2 \Rightarrow \hat{\omega}_1 > \hat{\omega}_2$ and environmental quality, $N_1 < N_2$. So, in case of two balanced growth rates with low growth, g_1, and high growth, g_2, the endogenous variables are ranked as $\rho_1 > \rho_2$, $\hat{\omega}_1 > \hat{\omega}_2$, $\hat{z}_1 < \hat{z}_2$, $N_1 < N_2$.

To examine stability, we compute the Jacobian Matrix of the three-dimensional dynamical system, (21)-(23). In matrix notation we get:

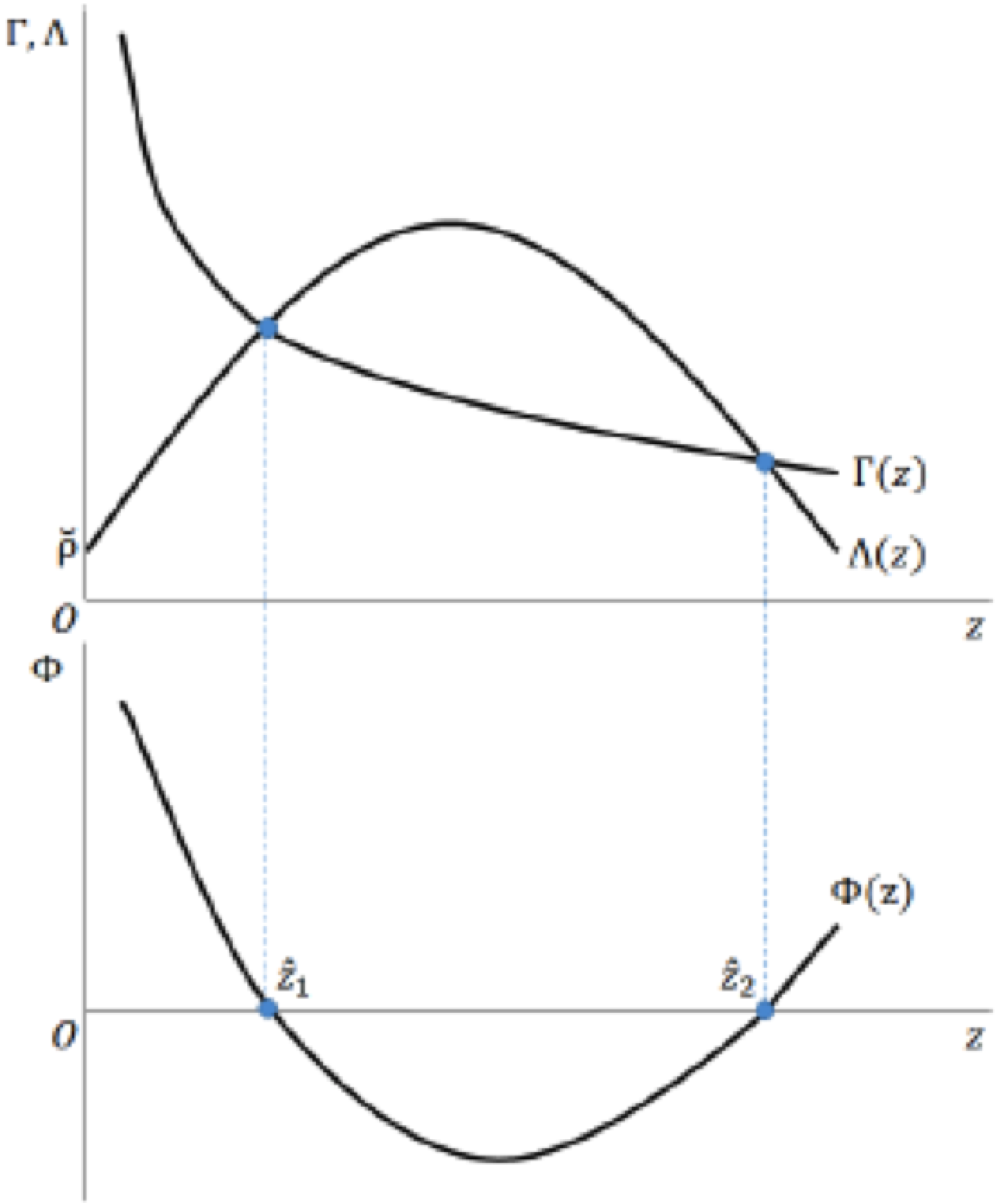
$$\begin{bmatrix} \dot{\omega} \\ \dot{z} \\ \dot{N} \end{bmatrix} = \Xi \begin{bmatrix} \omega - \bar{\omega} \\ z - \bar{z} \\ N - \bar{N} \end{bmatrix}$$

where $\Xi \equiv \begin{bmatrix} Jww & Jwz & JwN \\ Jzw & Jzz & JzN \\ JNw & JNz & JNN \end{bmatrix}$

and the elements of the matrix are given by: $Jww = \omega$, $Jwz = A(a-1)(1-\tau)(a-1)z^{a-2}\omega$, $JwN = -\rho^{/}(N)\omega$, $Jzz = A(1-\tau)(a-1)z^{a-1} - Ab\tau a z^{a}$, $Jzw = -z$, $JzN = 0$, $JNN = -(1-\delta_N)N$, $JNz = -(\frac{s}{\xi(1-b)\tau})N\omega$, $JNw = -(\frac{s}{\xi(1-b)\tau})Nz$.

The sign of J is ambiguous and depends on the parameters of the economy and the endogeneity of the rate of time preference as it affects JwN. The local dynamics of the economy are nontrivial and analytically intractable. For that reason we resort to numerical simulations, assuming standard parameter values of the growth literature, to (i) compute the two equilibria, (ii) solve for the characteristic equation and compute the eigenvalues of Ξ for those equilibria.

In particular, using the set of parameter values used in the paper ($\alpha = 0.5$, $A = 0.65$, $\delta = 0.14$, $\tau = 0.561$, $b = 0.751$, $\delta_N = 0.9$, $\xi = 0.4$, $s = 1$, $\bar{N} = 20$, $\gamma = 1$, $\breve{\rho} = 0.2$), we obtain for the low-growth and bad-environment equilibrium ($g_1 = 0.010$, $\hat{N}_1 = 0.089$) the following eigenvalues: $\varepsilon_1 = 0.295$, $\varepsilon_2 = -0.009$, $\varepsilon_3 = -0.259$ and for the high-growth and good-environment equilibrium ($g_2 = 0.035$, $\hat{N}_2 = 0.151$) the following eigenvalues: $\varepsilon_1 = 0.207$, $\varepsilon_2 = -0.015, \varepsilon_3 = -0.270$. Given that there exist two predetermined and one jump variables in our dynamical system, it follows that both equilibria are saddle-path stable. Thus, there exist parameter values such that the multiple (two) equilibria derived in Proposition 1 are both stable and, hence, meaningful for examining changes in the structural and policy parameters of the model.



A1.png

Figure 1: Graphical illustration of multiple log-run equilibria

Appendix B: Proof of Proposition 2

The task is to calculate the effect of A on $\hat{N}_1$, $\frac{\partial \hat{N}_1}{\partial A}$, under the presence of multiple equilibria, that is, $Aa(1-\tau)(\frac{1-\tau}{b\tau})^{a-1} - Ab\tau(\frac{1-\tau}{b\tau})^{a} > \check{\rho}$ and for the low equlibrium where $\hat{z}_1 < \frac{a(1-\tau)}{b(1+a)\tau}$.

From the above analysis it follows that at the low equilibrium, $\hat{z} < \frac{a(1-\tau)}{b(1+a)\tau}$, $\frac{\partial \Lambda(\hat{z})}{\partial z} = \acute{\rho}(\hat{N})\frac{\partial N}{\partial z} > 0$ as $\frac{\partial N}{\partial z} = -\Xi\frac{\partial(\hat{\omega}\hat{z})}{\partial z} < 0$ because $\frac{\partial(\hat{\omega}\hat{z})}{\partial z} > 0$.

The effect of productivity on environmental quality is given by:

$\frac{\partial \hat{N}_1}{\partial A} = -\Xi[\frac{\hat{\omega}_1\hat{z}_1}{A}] - \Xi\frac{\partial(\hat{\omega}_1\hat{z}_1)}{\partial z}\frac{\partial \hat{z}_1}{\partial A}$

At the low growth equilibrium, we have that $\hat{z} < \frac{a(1-\tau)}{b(1+a)\tau}$ and $\frac{\partial(\hat{\omega}\hat{z})}{\partial z} > 0$. So, if $\frac{\partial \hat{z}_1}{\partial A} < 0$, then it will follow that $\frac{\partial \hat{N}_1}{\partial A} < 0$. To examine $\frac{\partial \hat{z}_1}{\partial A}$ we need $\frac{\partial \hat{z}_1}{\partial A} = -\frac{\frac{\partial \Phi(\hat{z})}{\partial A}}{\frac{\partial \Phi(\hat{z})}{\partial \hat{z}}}$.

Then, it holds that $\frac{\partial \Phi(\hat{z})}{\partial \hat{z}} = \underbrace{-Ab\tau a\hat{z}^{a-1} + Aa(a-1)(1-\tau)\hat{z}^{a-2}}_{<0} - \acute{\rho}(\hat{N})\frac{\partial N}{\partial z}$.

Also, we have that $\frac{\partial \Phi(\hat{z})}{\partial A} = \underbrace{-b\tau\hat{z}^{a} + a(1-\tau)\hat{z}^{a-1}}_{>0} + \acute{\rho}(\hat{N})\Xi(\frac{\hat{\omega}\hat{z}}{A})$.

Obviously, under exogenous rate of time preference, an increase in A increases the physical to public capital ratio. For endogenous rate of time preference, at the low equlbrium where $\hat{z} < \frac{a(1-\tau)}{b(1+a)\tau}$, $\acute{\rho}(\hat{N})\frac{\partial N}{\partial z} < 0$, thus $\frac{\partial \Phi(\hat{z})}{\partial \hat{z}} < 0$.

Then, $\frac{\partial \Phi(\hat{z}_1)}{\partial A} < 0$ if $-b\tau\hat{z}^{a} + a(1-\tau)\hat{z}^{a-1} + \acute{\rho}(\hat{N})\Xi\left[(1-\tau)\hat{z}^{a} - b\tau\hat{z}^{a+1}\right] < 0 \Rightarrow$

$-b\tau + a(1-\tau)\hat{z}^{-1} + \acute{\rho}(\hat{N})\Xi\left[(1-\tau) - b\tau\hat{z}^{1}\right] < 0 \Rightarrow -\acute{\rho}(\hat{N}) > \frac{-b\tau + a(1-\tau)\hat{z}^{-1}}{\Xi[(1-\tau)-b\tau\hat{z}^{1}]}$ which holds if the responsiveness of time preference is high. Then, for parameter values that satisfy $-\acute{\rho}(\hat{N}) > \frac{-b\tau + a(1-\tau)\hat{z}^{-1}}{\Xi[(1-\tau)-b\tau\hat{z}^{1}]}$, it follows that $\frac{\partial \hat{z}_1}{\partial A} < 0$. In Table 1 we show that such parameter values exist. Then, $\frac{\partial \hat{N}_1}{\partial A} = -\Xi[\frac{\hat{\omega}_1\hat{z}_1}{A}] - \Xi\frac{\partial(\hat{\omega}_1\hat{z}_1)}{\partial z}\frac{\partial \hat{z}_1}{\partial A} < 0$ and, since, $\frac{\partial \hat{z}_1}{\partial A} < 0$, it is straightforward to show that $\frac{\partial \hat{\omega}_1}{\partial A} > 0$ and $\frac{\partial g}{\partial A} < 0$.