Fiscal Consolidation in a Low Inflation Environment: Pay Cuts versus Lost Jobs

Guilherme Bandeira Evi Pappa Rana Sajedi Eugenia Vella

November 2017

Online Appendix

1 Model Derivations

1.1 Household's maximisation problem

The household's Lagrangean can be written as

$$\begin{aligned}
\mathcal{L} &= \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(c_t - h_b c_{t-1})^{1-\eta}}{1-\eta} + \Phi \frac{(1 - n_t^p - n_t^g - u_t)_t^{1+\varphi}}{1+\varphi} \right. \\
&- \lambda_{c,t} \left[(1+\tau_c)\, c_t + i_t + b_{g,t+1} + e_t r_{f,t-1} b_{f,t} - \left[r_t^k - \tau_k \left(r_t^k - \bar{\delta} x_t^{\xi} \right) \right] x_t k_t^p \right. \\
&\quad \left. - r_{t-1} b_{g,t} - e_t b_{f,t+1} - (1-\tau_n)\left(w_t^p n_t^p + w_t^g n_t^g \right) - \mathrm{b} u_t - \Pi_t^p - T_t \right] \\
&- \lambda_{k,t} \left[k_{t+1}^p - \left[1 - \frac{\omega}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t - \left(1 - \bar{\delta} x_t^{\xi} \right) k_t^p \right] \\
&- \lambda_{n^p,t} \left[n_{t+1}^p - (1-\sigma^p) n_t^p - \psi_t^{hp} (1 - s_t) u_t \right] \\
&- \left. \lambda_{n^g,t} \left[n_{t+1}^g - (1-\sigma^g) n_t^g - \psi_t^{hg} s_t u_t \right] \right\}
\end{aligned}$$

where the household's composition has been substituted into the utility function, and the household takes as given the probability of finding a job in each sector and does not internalise the effect of their choice of u_t^jon the number of matches. We assume external habits in consumption, meaning that c_{t-1} is taken as given in period t.

The choice variables are c_t, i_t, k_{t+1}^p, x_t, n_{t+1}^p, n_{t+1}^g, u_t, s_t, $b_{g,t+1}$ and $b_{f,t+1}$. The first order conditions are:

[wrt c_t]

$$\lambda_{c,t} (1+\tau_c) = (c_t - h_b c_{t-1})^{-\eta} \tag{1}$$

[wrt i_t]

$$\lambda_{c,t} - \lambda_{k,t} \left\{ 1 - \frac{\omega}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \omega \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right\} = \beta \lambda_{k,t+1} \omega \left(\frac{i_{t+1}}{i_t} - 1 \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \tag{2}$$

$\left[\text{wrt } k^p_{t+1}\right]$

$$\lambda_{k,t} = \beta\left\{\lambda_{k,t+1}\left(1-\bar{\delta}x_t^{\xi}\right)+\lambda_{c,t+1}\left[r^k_{t+1}-\tau_k\left(r^k_{t+1}-\bar{\delta}x_t^{\xi}\right)\right]\right\} \tag{3}$$

$[\text{wrt } x_t]$

$$r^k_t-\tau_k(r^k_t-\bar{\delta}x_t^{\xi})+\tau_k\xi\bar{\delta}x_t^{\xi}=\bar{\delta}\xi x_t^{\xi-1} \tag{4}$$

$\left[\text{wrt } n^p_{t+1}\right]$

$$\lambda_{n^p,t} = \beta\left[\lambda_{n^p,t+1}\left(1-\sigma^p\right)+\lambda_{c,t+1}\left(1-\tau_n\right)w^p_{t+1}-\Phi l^{\varphi}_{t+1}\right] \tag{5}$$

$\left[\text{wrt } n^g_{t+1}\right]$

$$\lambda_{n^g,t} = \beta\left[\lambda_{n^g,t+1}\left(1-\sigma^g\right)+\lambda_{c,t+1}\left(1-\tau_n\right)w^g_{t+1}-\Phi l^{\varphi}_{t+1}\right] \tag{6}$$

$[\text{wrt } u_t]$

$$\Phi l^{\varphi}_t = \lambda_{c,t}\mathrm{b}+\lambda_{n^p,t}\psi^{hp}_t\left(1-s_t\right)+\lambda_{n^g,t}\psi^{hg}_t s_t \tag{7}$$

$[\text{wrt } s_t]$

$$\lambda_{n^g,t}\psi^{hg}_t = \lambda_{n^p,t}\psi^{hp}_t \tag{8}$$

$[\text{wrt } b_{g,t+1}]$

$$1 = \beta\frac{\lambda_{c,t+1}}{\lambda_{c,t}}r_t \tag{9}$$

[wrt $b_{f,t+1}$]

$$1 = \beta \frac{\lambda_{c,t+1}}{\lambda_{c,t}} \frac{e_{t+1}}{e_t} r_{f,t} \tag{10}$$

We can define the marginal value to the household of having an additional member employed in the private sector, as follows:

$$\begin{aligned} V^h_{n^p t} \equiv \frac{\partial \mathcal{L}}{\partial n^p_t} &= \lambda_{c,t} w^p_t (1-\tau_n) - \Phi l^{\varphi}_t + (1-\sigma^p)\lambda_{n^p t} \\ &= \lambda_{ct} w^p_t (1-\tau_n) - \Phi l^{\varphi}_t + (1-\sigma^p)\beta E_t(V^h_{n^p t+1}) \end{aligned} \tag{11}$$

where the second equalities come from equation (5).

1.2 Derivation of the private wage

The Nash bargaining problem is to maximize the weighted sum of log surpluses:

$$\max_{w^p_t} \left\{ (1-\vartheta) \ln V^h_{n^p t} + \vartheta \ln V^f_{n^p t} \right\}$$

where $V^h_{n^j t}$ and $V^f_{n^j t}$ are defined as:

$$V^h_{n^p t} \equiv \frac{\partial \mathcal{L}}{\partial n^p_t} = \lambda_{ct} w^p_t (1-\tau^n_t) - \Phi l^{\varphi}_t + (1-\sigma^p)\lambda_{n^p t} \tag{12}$$

$$V^F_{n^p t} \equiv \frac{\partial Q^p}{\partial n^p_t} = p_{x,t}(1-\phi)\frac{y^p_t}{n^p_t} - w^p_t + \frac{(1-\sigma^p)\kappa}{\psi^{fp}_t} \tag{13}$$

The first order conditions of this optimization problem is:

$$\vartheta V^h_{n^p t} = (1-\vartheta)\lambda_{ct}(1-\tau^n_t)V^f_{n^p t} \tag{14}$$

Plugging the expressions for the value functions into the FOC, we can rearrange to find the

expression for the private wage. Using (12),(13) and (14) we obtain:

$$w_t^p = (1-\vartheta)[p_{x,t}(1-\phi)\frac{y_t^p}{n_t^p} + \frac{(1-\sigma^p)\kappa}{\psi_t^{fp}}] + \frac{\vartheta}{(1-\tau_n)\lambda_{c,t}}(\Phi l_t^{\varphi} - (1-\sigma^p)\lambda_{n^p t}) \tag{15}$$

Finally, taking the time t expectation of (14) evaluated at time $t+1$, and using the FOCs of the household and firm, we obtain

$$\vartheta\lambda_{n^p t} = (1-\vartheta)\lambda_{ct}(1-\tau_t^n)\frac{\kappa}{\psi_t^{fp}}$$

which allows us to simplify (15) to obtain the final expression for the private wage

$$w_t^p = (1-\vartheta)p_{x,t}(1-\phi)\frac{y_t^p}{n_t^p} + \frac{\vartheta}{(1-\tau_n)\lambda_{c,t}}\Phi l_t^{\varphi} \tag{16}$$

2 Additional Figures

Here we include the following figures:

- The effects of consolidation on total employment
- Consolidation in normal times without variable capital utilization
- Consolidation at the ZLB without the shock to the relative price of investment
- Consolidation at the ZLB with utility-enhancing public good
- Consolidation at the ZLB with sticky wages[1]
- Sensitivity to asymmetric labor immobility

[1]Specifically, we assume that private real wages evolve according to Monacelli et al. (2010):

$$w_t^p = \rho_w w_{t-1}^p + (1-\rho_w)\left\{\left((1-\vartheta)p_{x,t}(1-\phi)\frac{y_t^p}{n_t^p} + \frac{\vartheta}{(1-\tau_t^n)\lambda_{c,t}}\Phi l_t^{-\varphi}\right)\right\} \tag{17}$$

where we set ρ_w, the degree of real wage rigidity, to 0.75. By definition, when private wages are sticky they move less in response to the shocks. This leads firms to cut private vacancies more sharply on impact, mitigating the reallocation of jobseekers towards the private sector. Consequently, public sector employment falls by less, which mitigates the effects on unemployment. Yet, the assumption of sticky wages does not seem to substantially affect any of our previous conclusions. Indeed, Krause et al. (2007) have shown that inflation dynamics are only weakly affected by real wage rigidity. This is because in a labor market with search and matching frictions, the cost of employing an additional worker is no longer just the wage, but the value of the long-term relationship that the firm and worker enjoy.

- Consolidation with higher public wage premium[2]

- Repeat of sensitivity analysis in the main text, showing results in normal times

[2] In the baseline calibration we assume that 1/5 of all jobseekers are searching in the public sector. This, together with the remaining calibration of the labor market, generates a public wage premium of $w^g/w^p - 1 = 1\%$. We increase this number to 5% by raising the share of jobseekers searching in the public sector to 26.2%.

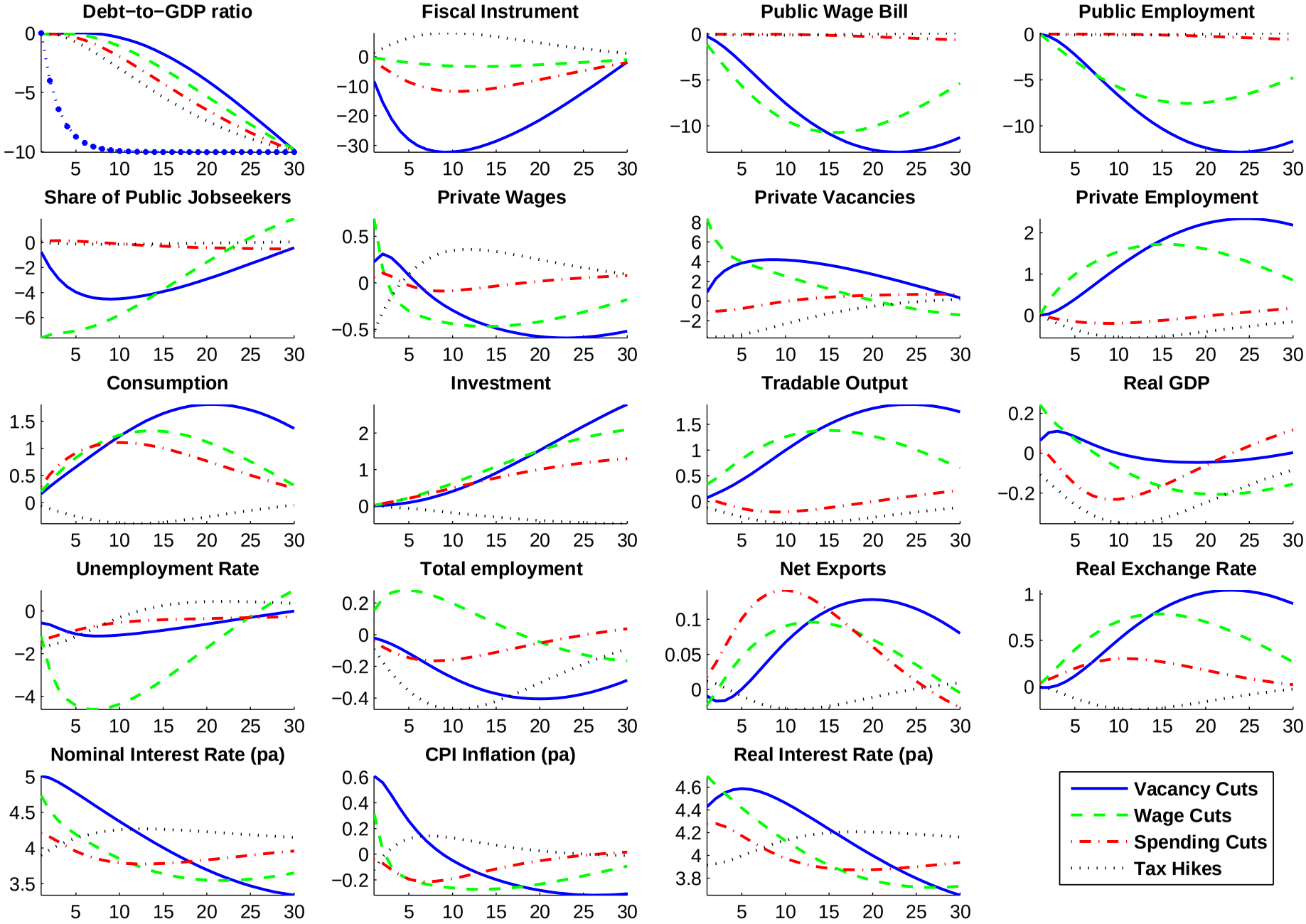


Figure 1: Normal Times: Effects on Total Employment

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is in percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.

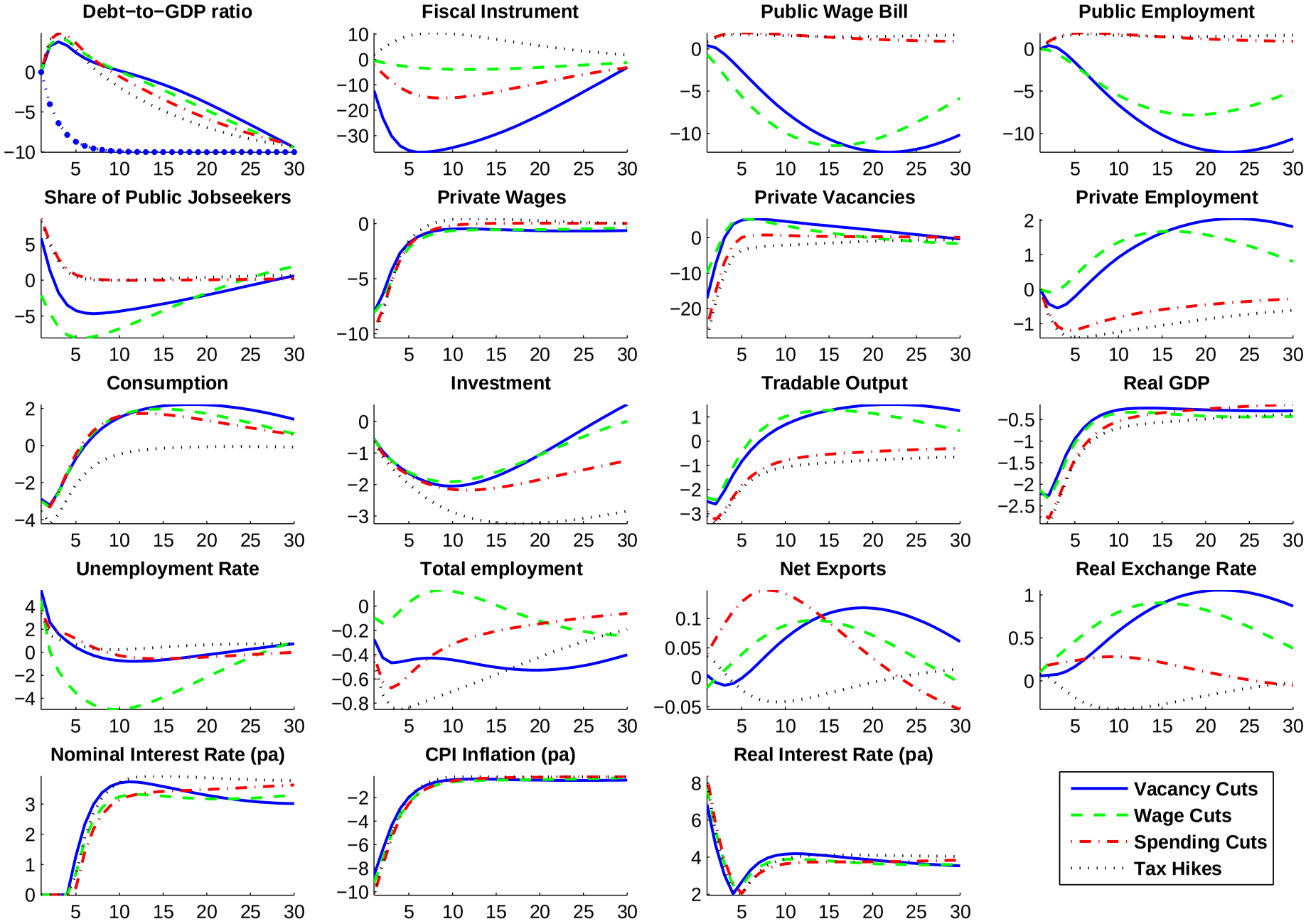
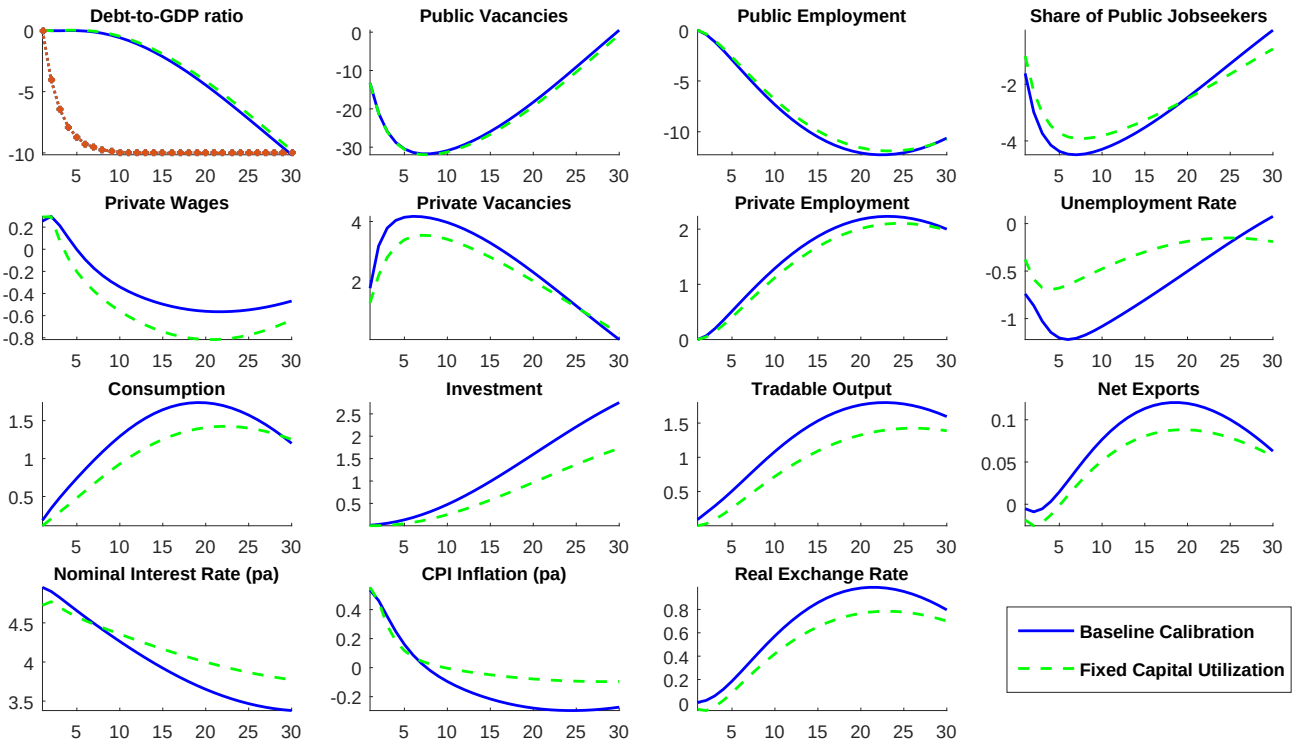


Figure 2: Low Inflation Environment: Effects on Total Employment

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is in percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.

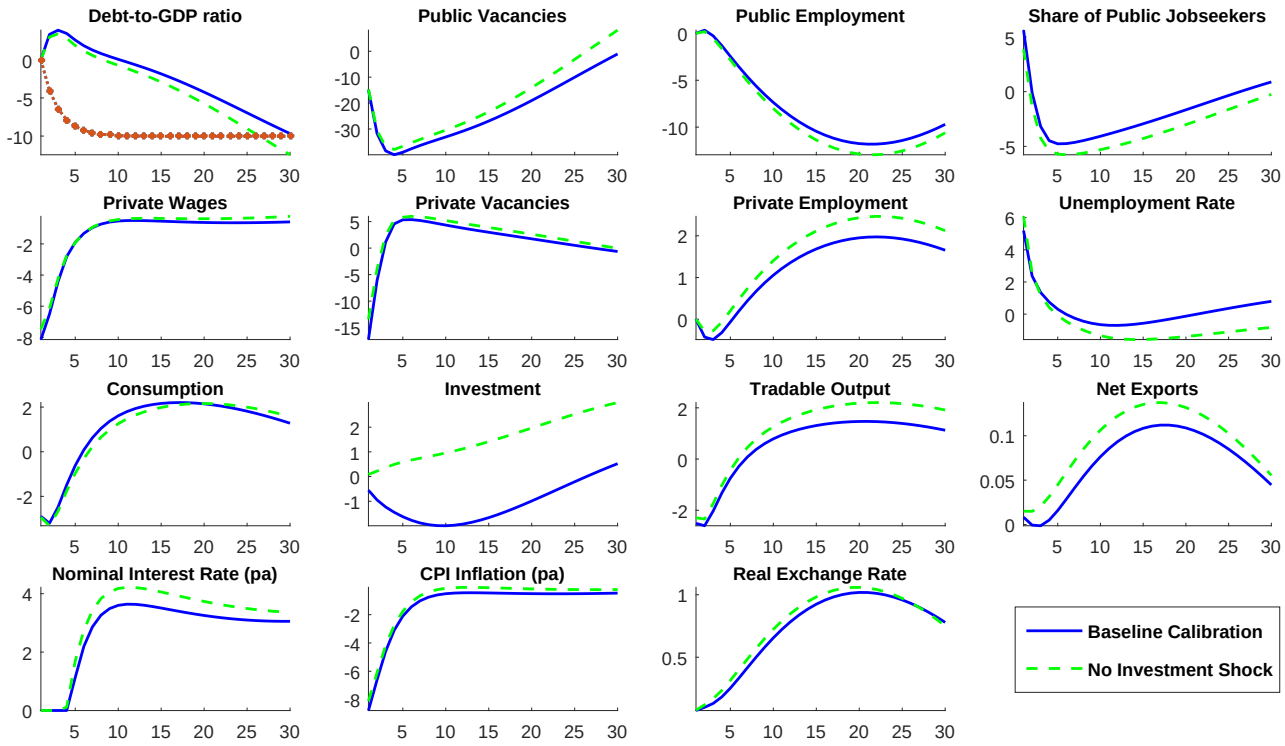


(a) Public Vacancy Cuts

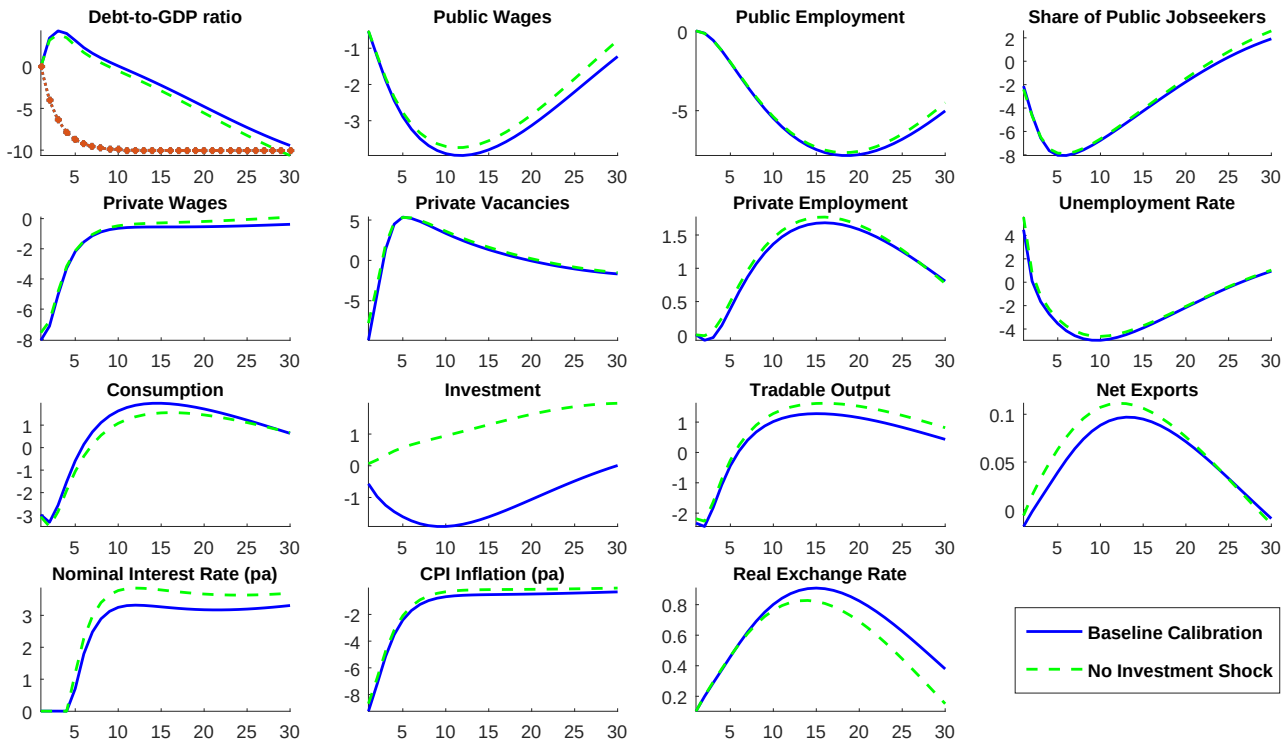
(b) Public Wage Cuts

Figure 3: Fiscal Consolidation in Normal Times Without Variable Capital Utilization

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.



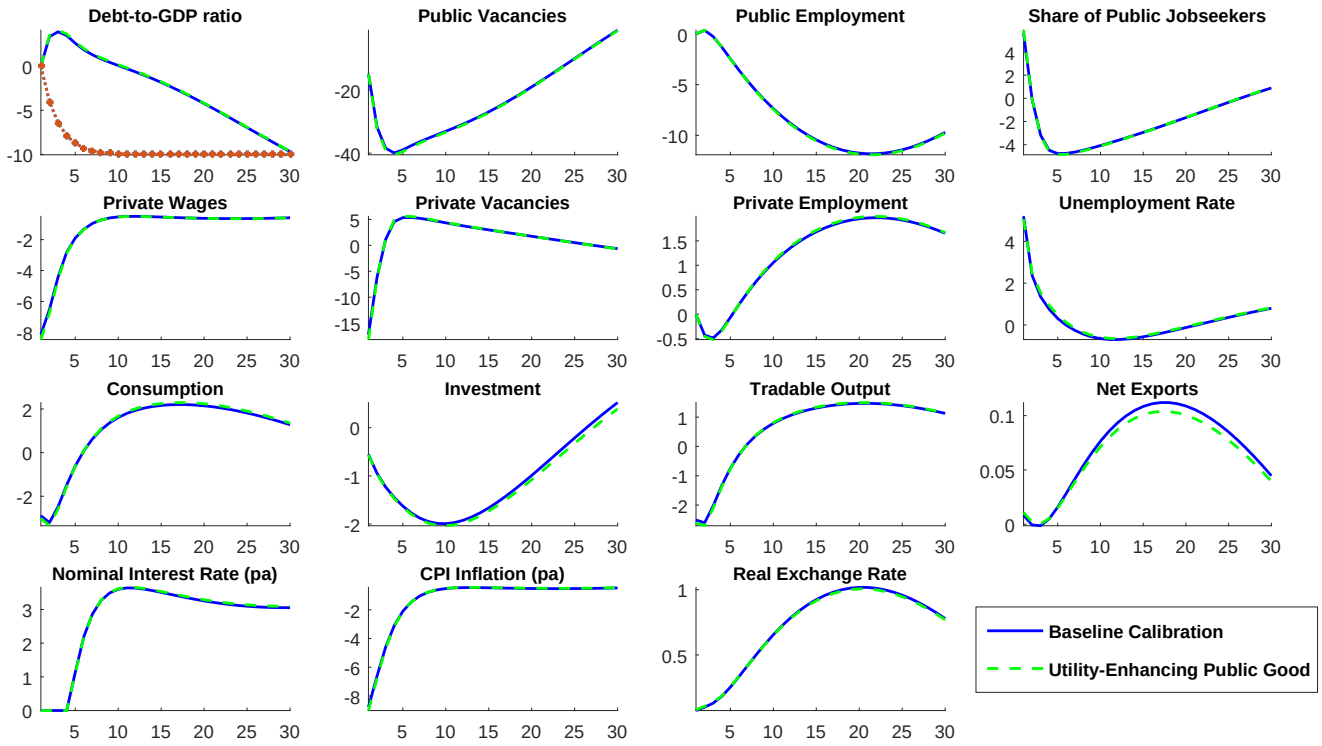
(a) Public Vacancy Cuts



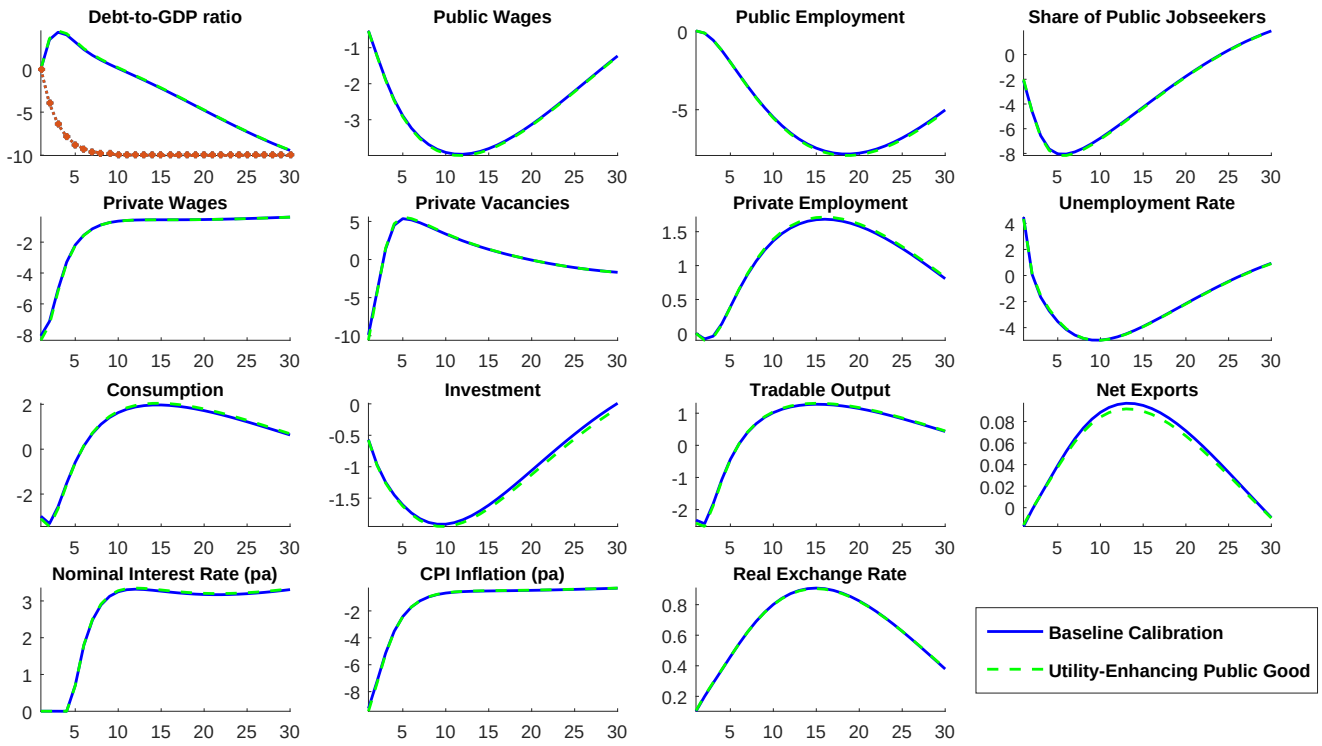
(b) Public Wage Cuts

Figure 4: Fiscal Consolidation at the ZLB Without Shock to Relative Price of Investment

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.



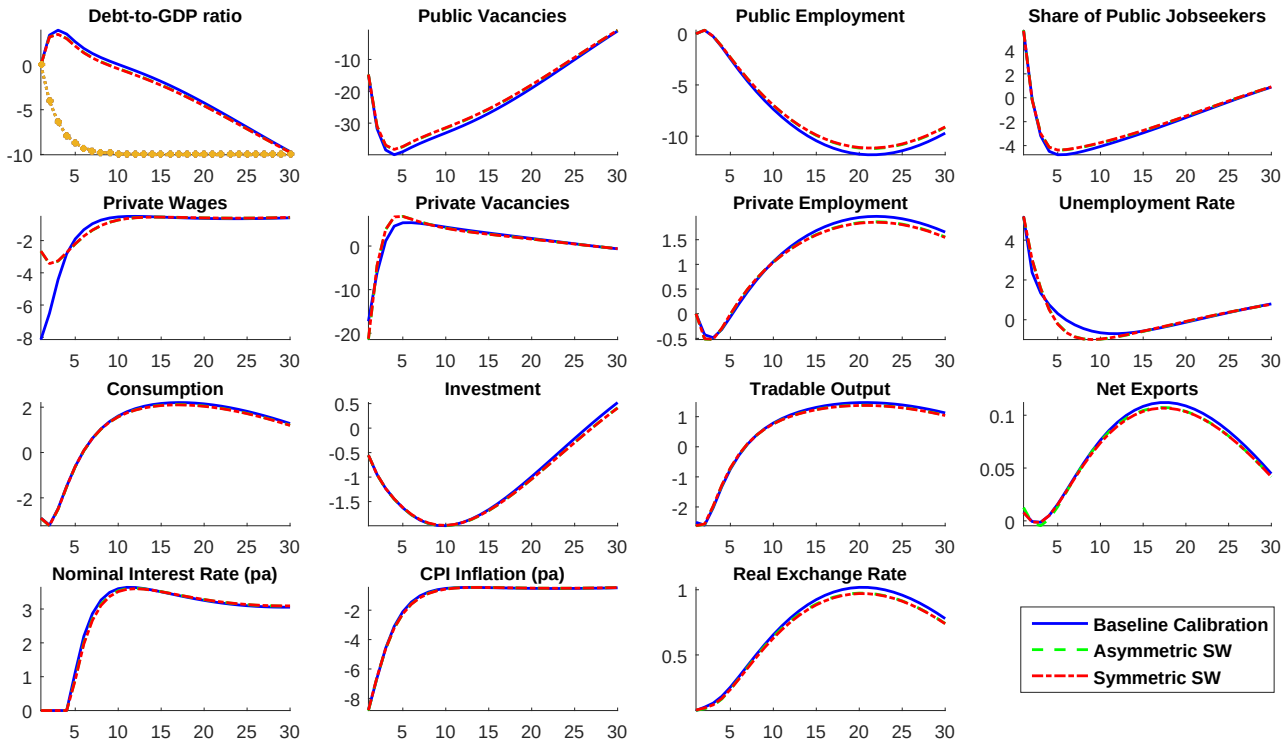
(a) Public Vacancy Cuts



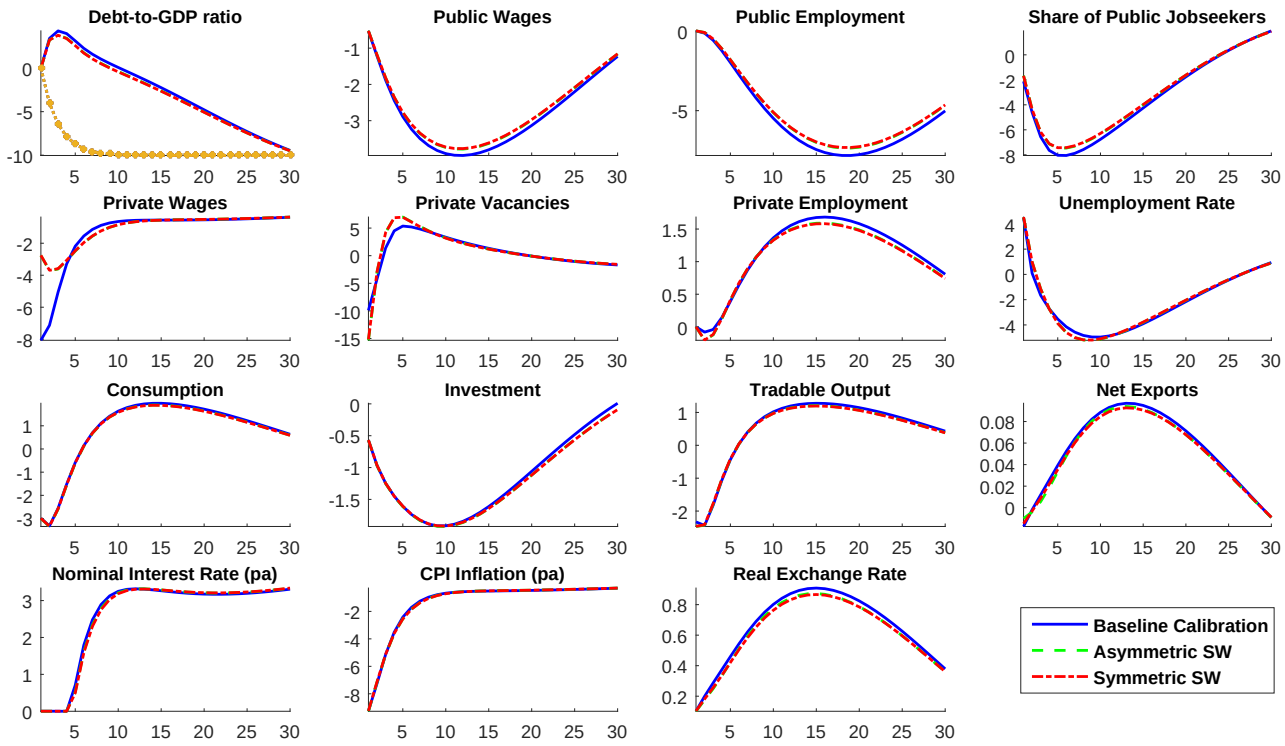
(b) Public Wage Cuts

Figure 5: Utility-Enhancing Public Good (UE) at the Zero Lower Bound

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.



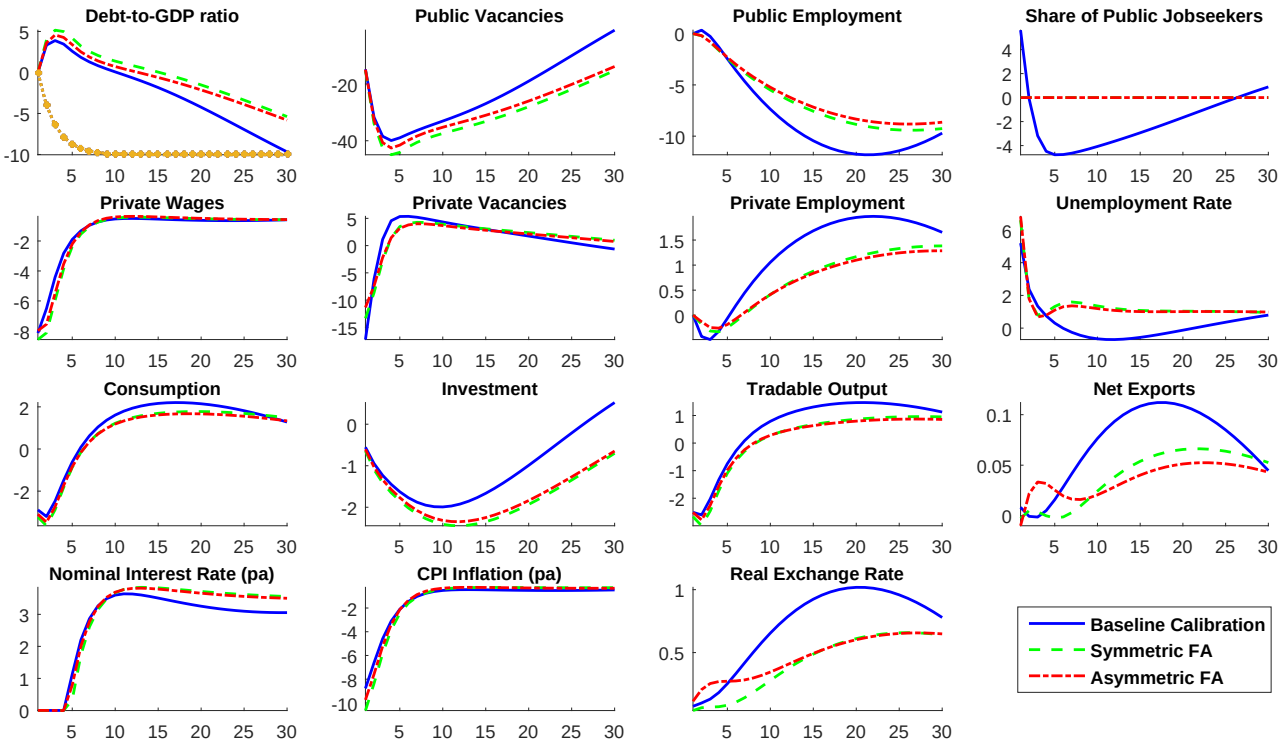
(a) Public Vacancy Cuts



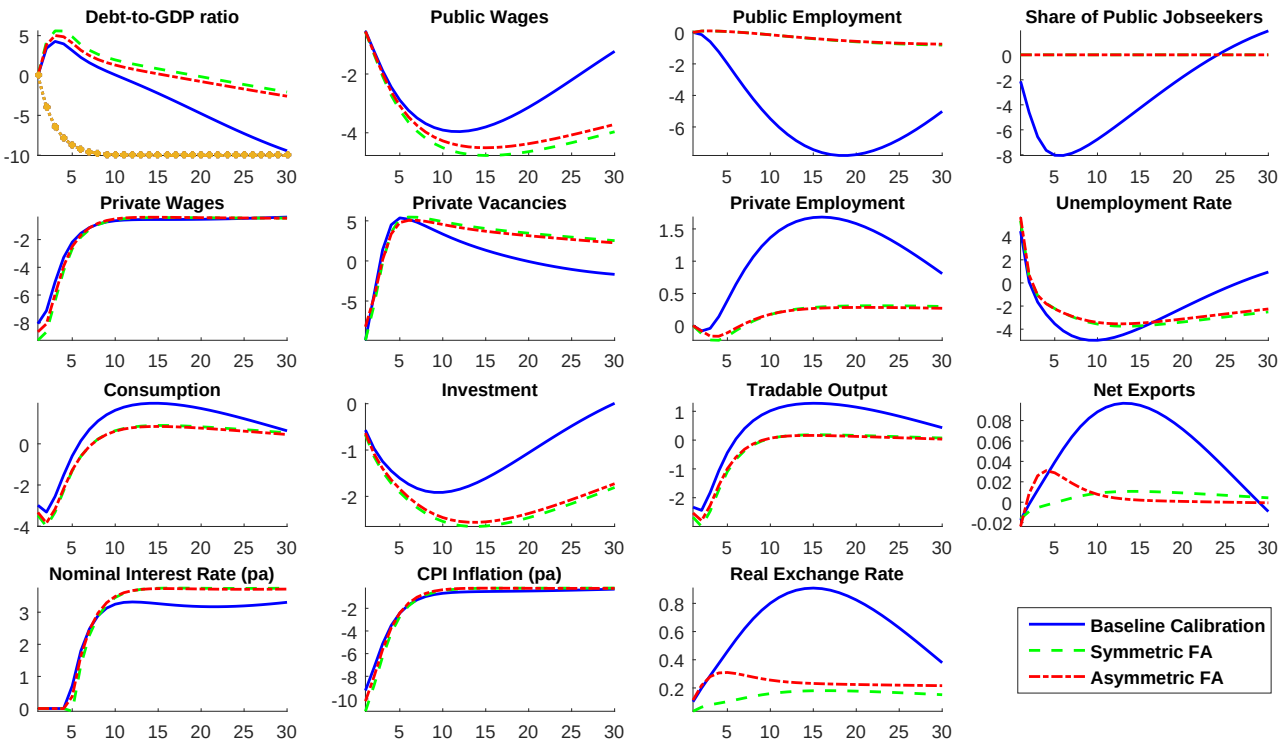
(b) Public Wage Cuts

Figure 6: Sticky Wages (SW) at the Zero Lower Bound

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.



(a) Public Vacancy Cuts



(b) Public Wage Cuts

Figure 7: Fixed Allocation of Jobseekers (FA) at the Zero Lower Bound

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.

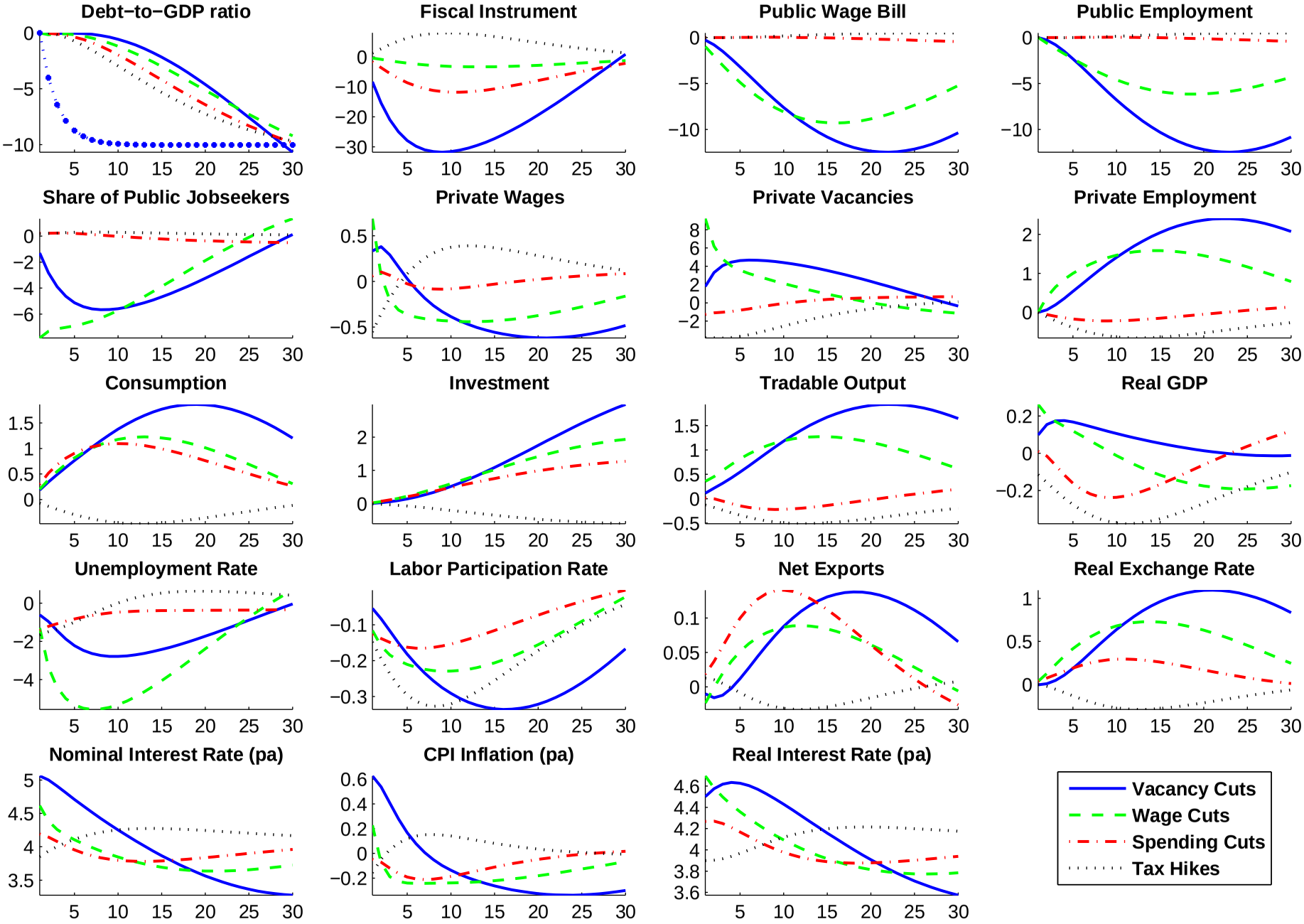


Figure 8: Normal Times: Higher Public Wage Premium

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is in percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.

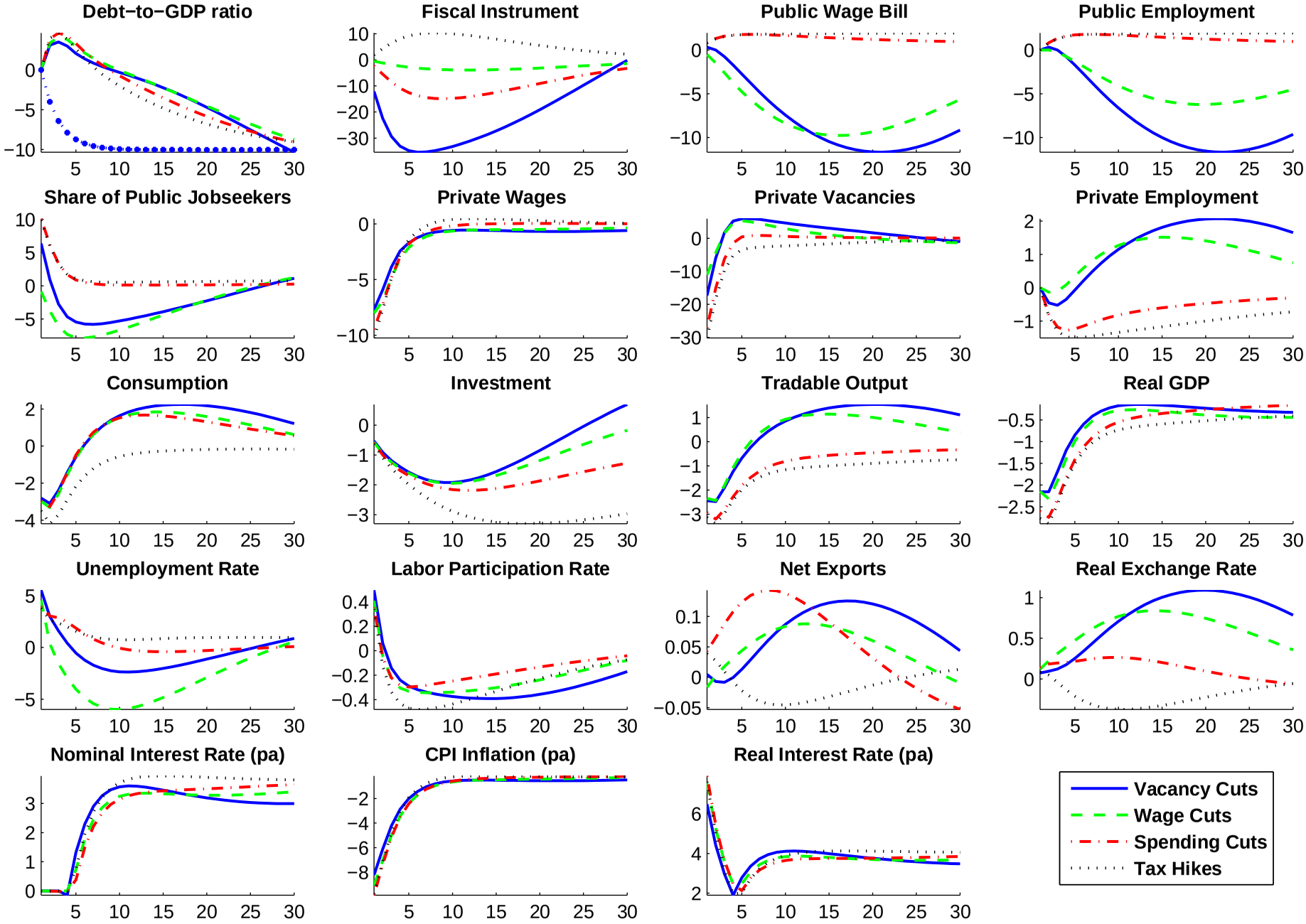
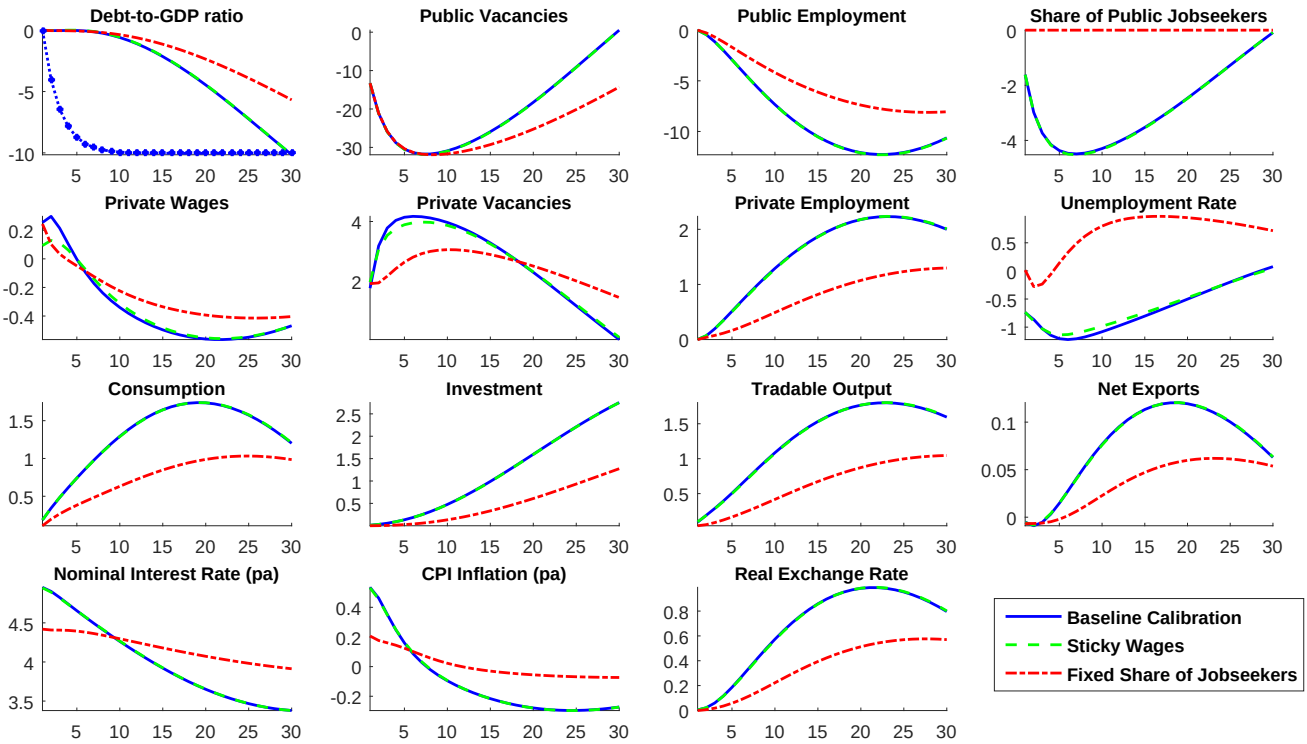
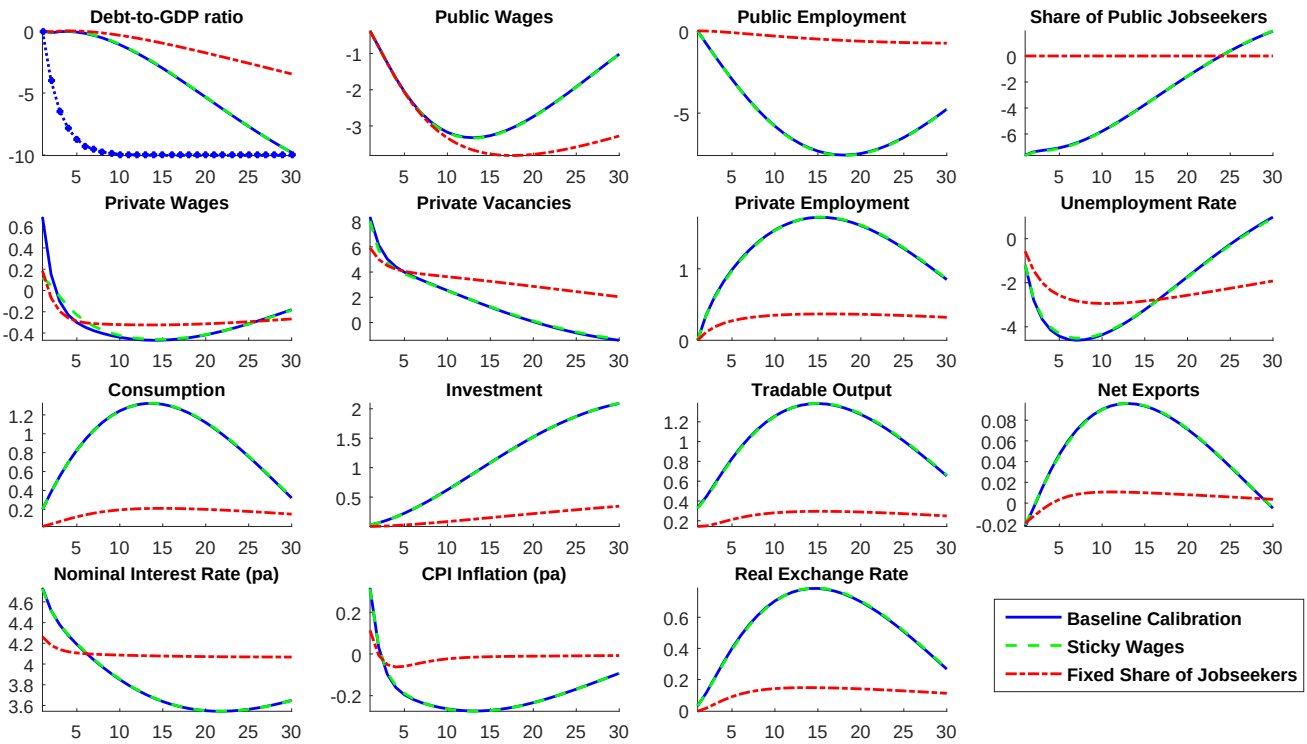


Figure 9: Low Inflation Environment: Higher Public Wage Premium

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is in percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.



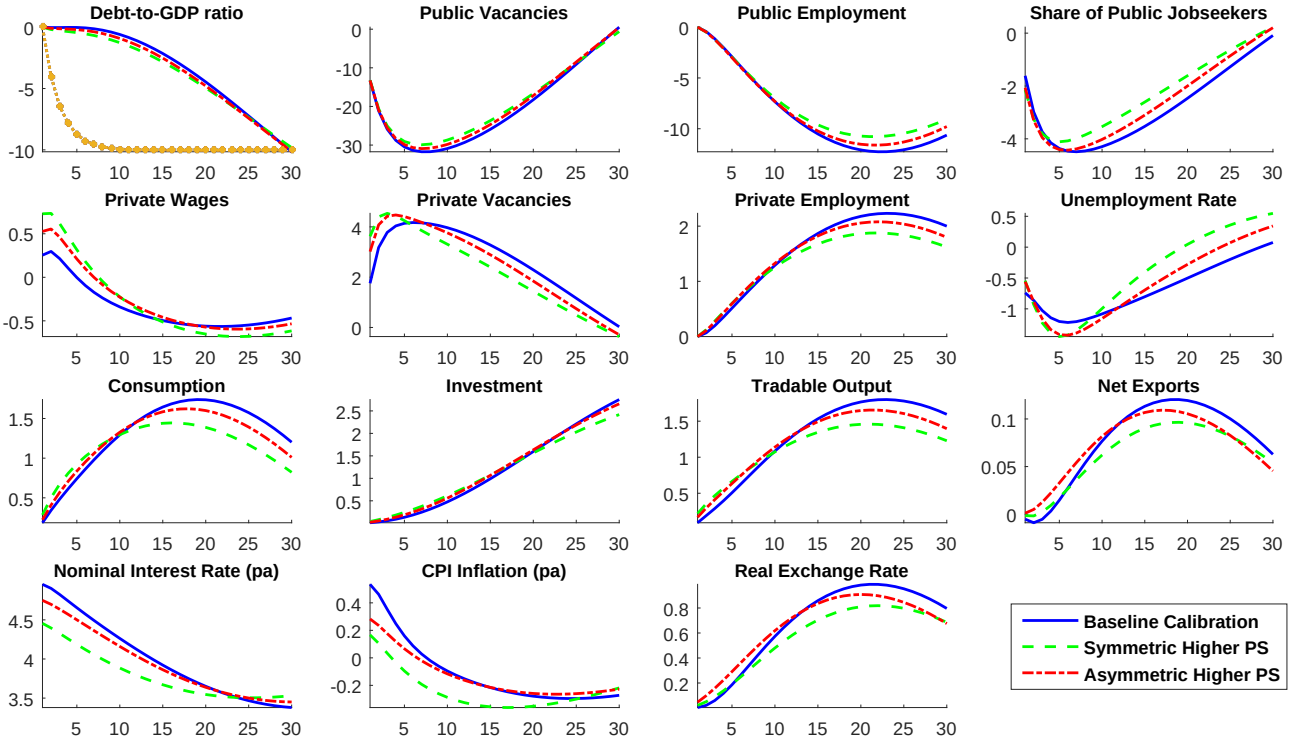
(a) Public Vacancy Cuts



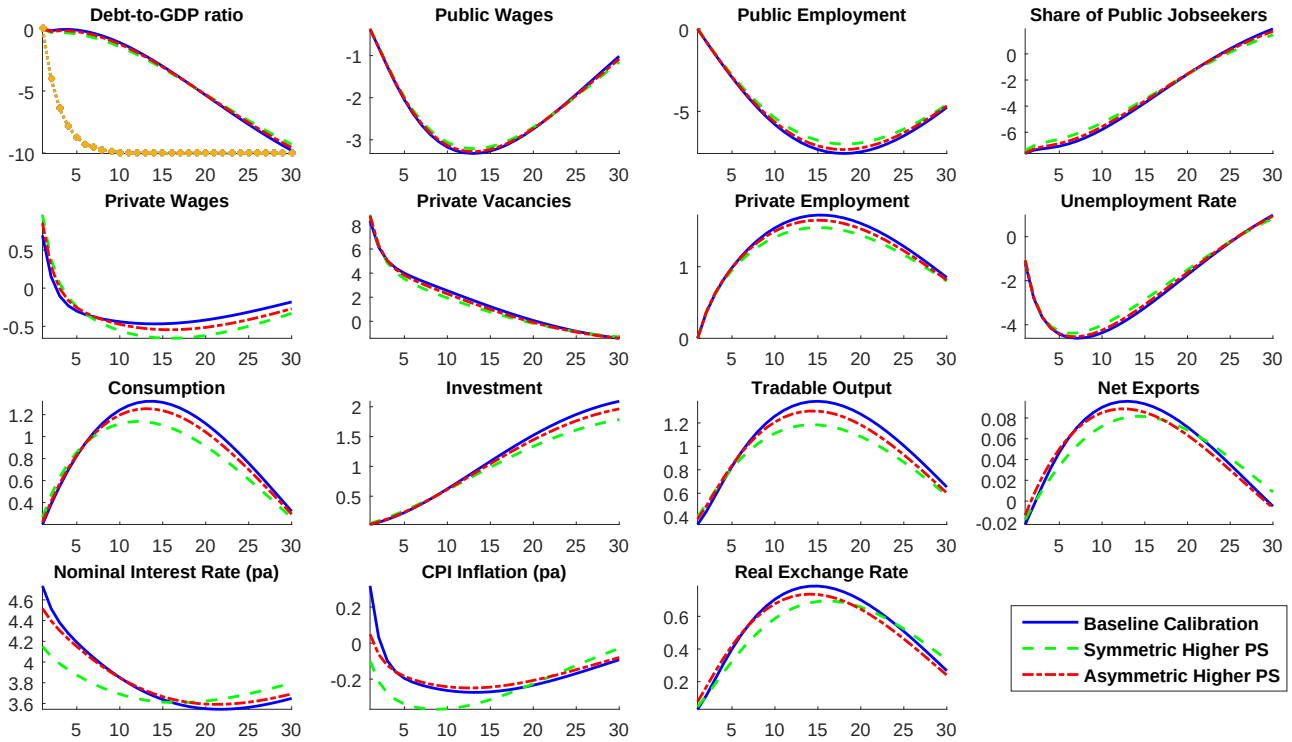
(b) Public Wage Cuts

Figure 10: Labour Market Rigidities in Normal Times

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.



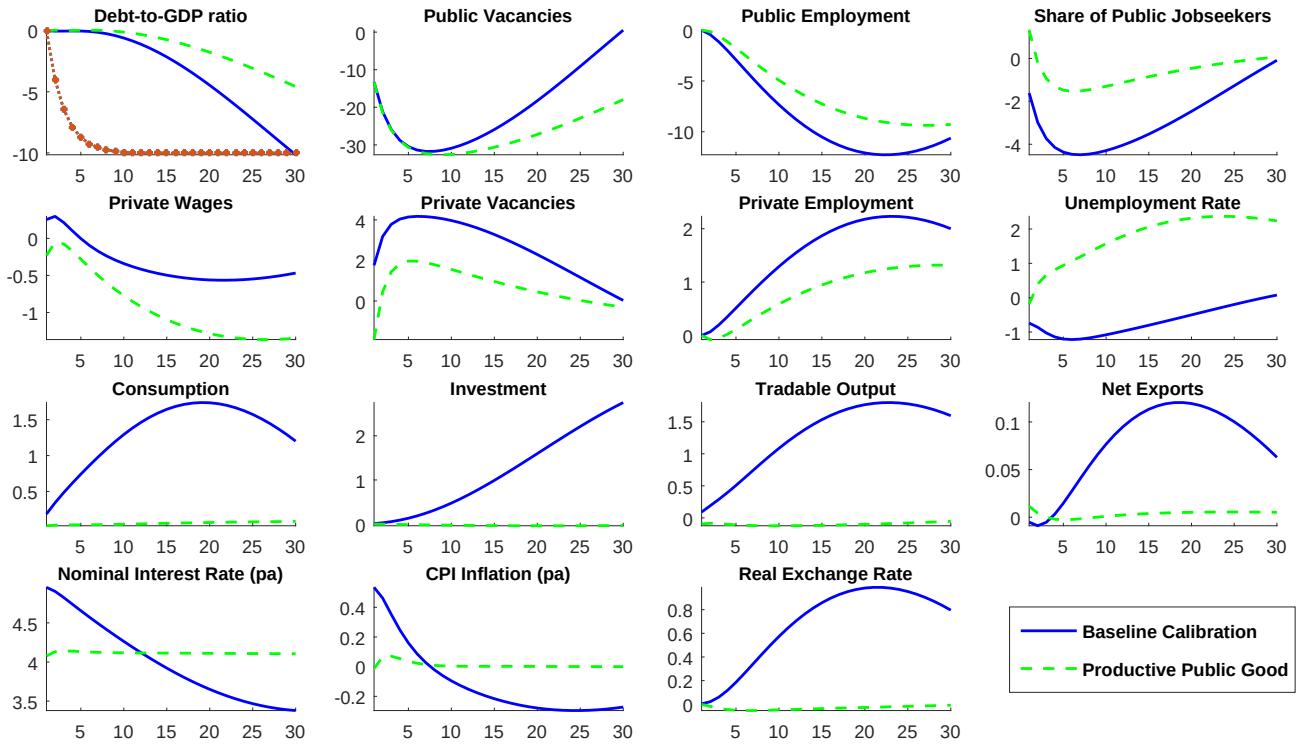
(a) Public Vacancy Cuts



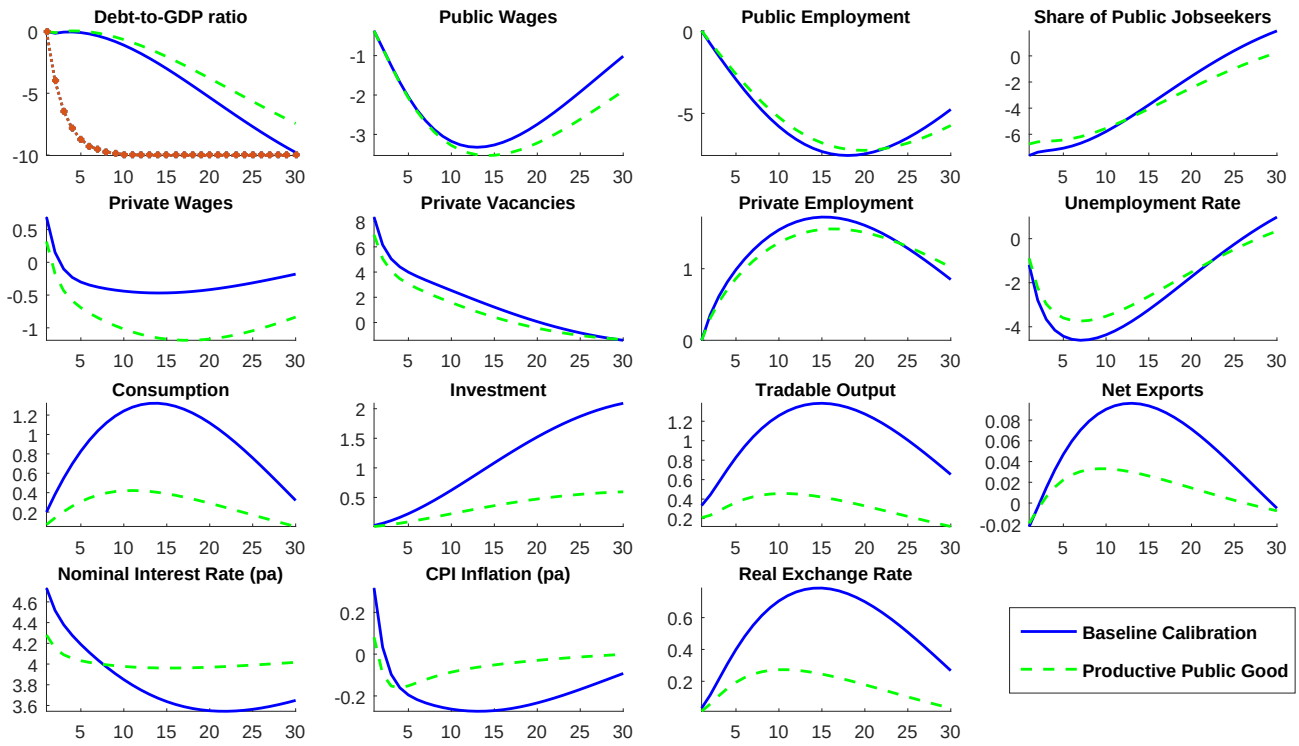
(b) Public Wage Cuts

Figure 11: Higher Price Stickiness (PS) in Normal Times

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.



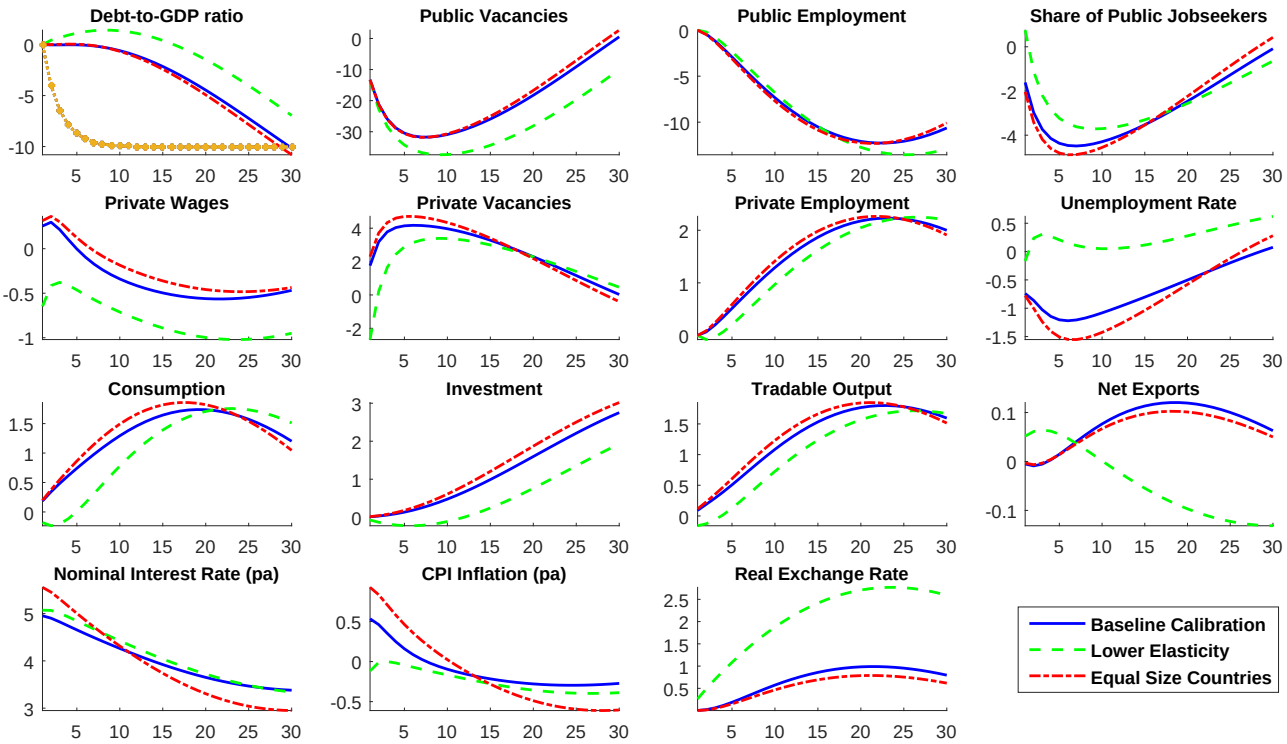
(a) Public Vacancy Cuts



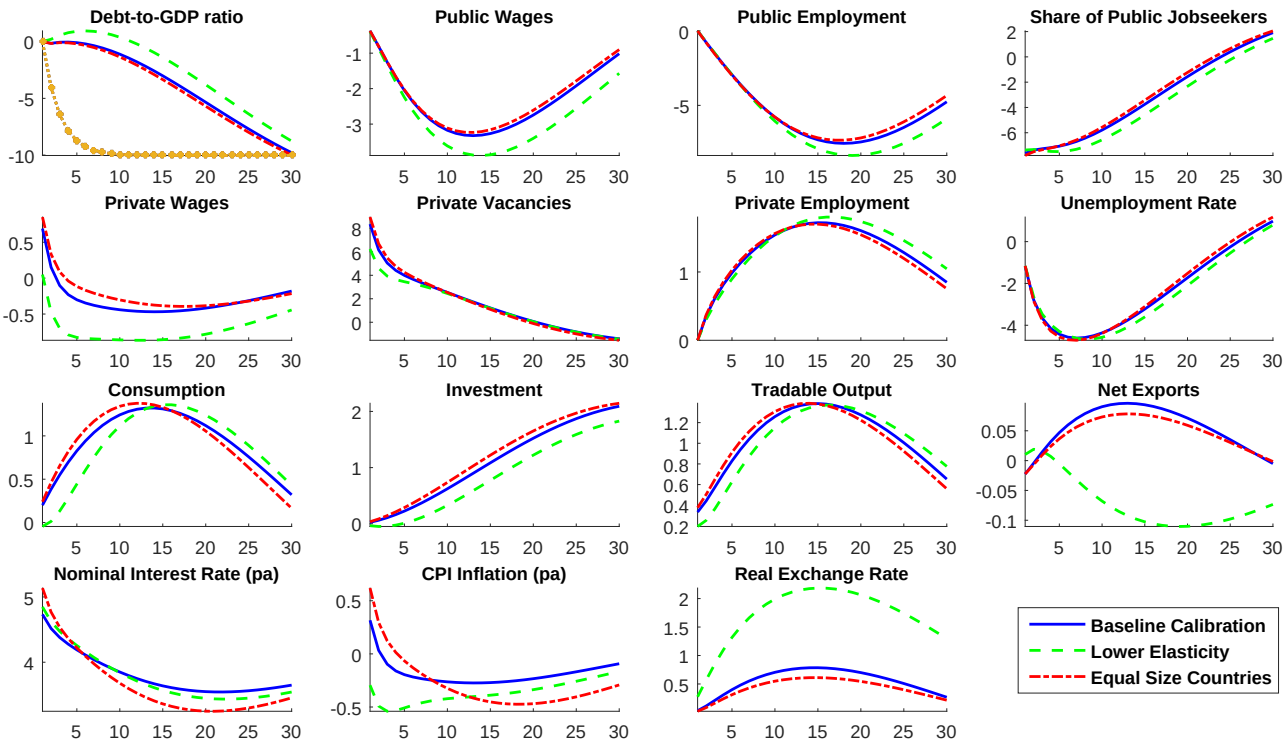
(b) Public Wage Cuts

Figure 12: Productive Public Good in Normal Times

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.



(a) Public Vacancy Cuts



(b) Public Wage Cuts

Figure 13: Open Economy Parameters in Normal Times

Responses are in percent deviations from steady state, except for interest rates and inflation, which are in annualized levels, the share of public jobseekers, which is percentage point deviation from steady state, and net exports, which are in levels. The line with diamond markers on the top-left panel reports the path for the debt-to-GDP target.