Companion Appendix to
“Green Spending Reforms, Growth and Welfare with Endogenous Subjective Discounting”

(Not intended for publication)

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1 Proof of Proposition 1

Let us first investigate the conditions for a well-defined equilibrium in the long run. In order for the balanced growth rate to be positive, we must have $\hat{z} > \left(\frac{\delta}{b\tau}\right)^{\frac{1}{a}}$ from (9c). Also, in order for $\hat{\omega}(\hat{z}) > 0$ and $\hat{x}(\hat{z}) > 0$ to hold, we must have $\hat{z} < \frac{1-\tau}{b\tau}$ from (12a) and $\hat{z} < \left(\frac{\delta+\delta_N}{b\tau A}\right)^{\frac{1}{a}}$ from (12b), since we are assuming that $\Theta(\tau, b) < 0$. Combining all the above we get the following for the domain of $\hat{z}$:

(i) if $\delta + \delta_N \geq A(1-\tau)^a(b\tau)^{1-a}$, then $\left(\frac{\delta}{b\tau}\right)^{\frac{1}{a}} < \hat{z} < \frac{1-\tau}{b\tau}$

(ii) if $\delta + \delta_N \leq A(1-\tau)^a(b\tau)^{1-a}$, then $\left(\frac{\delta}{b\tau}\right)^{\frac{1}{a}} < \hat{z} < \left(\frac{\delta+\delta_N}{b\tau A}\right)^{\frac{1}{a}}$.

The next step is to solve (12c) by separating function $\Phi(z)$ in two parts and finding their intersection. We thus define $\Gamma(z) \equiv -\sigma b\tau A z^a + a(1-\tau)Az^{a-1} - (1-\sigma)\delta$ and $\Lambda(z) \equiv \rho(z \cdot \omega(z) \cdot x(z))$. $\Gamma(z)$ has the following properties:

1. $\Gamma(z)$ is continuous in z.
2. $\lim_{z \to \left(\frac{\delta}{b\tau}\right)^{\frac{1}{a}}} \Gamma(z) = a(1-\tau)A\left(\frac{\delta}{b\tau}\right)^{\frac{a-1}{a}} - (1-\sigma+\sigma A)\delta.$
3. $\lim_{z \to \frac{1-\tau}{b\tau}} \Gamma(z) = A(a-\sigma)(1-\tau)^a(b\tau)^{1-a} - (1-\sigma)\delta.$
4. $\lim_{z \to \left(\frac{\delta+\delta_N}{b\tau A}\right)^{\frac{1}{a}}} \Gamma(z) = -\sigma\delta_N - \delta + a(1-\tau)A\left(\frac{\delta+\delta_N}{b\tau A}\right)^{-\frac{(1-a)}{a}}.$
5. $\frac{\partial \Gamma(z)}{\partial z} = -a\sigma b\tau A z^{a-1} - (1-a)a(1-\tau)Az^{a-2} < 0.$
6. $\frac{\partial^2 \Gamma(z)}{\partial z^2} = (1-a)a\sigma b\tau A z^{a-2} + (2-a)(1-a)a(1-\tau)Az^{a-3} > 0.$

In turn, $\Lambda(z)$ has the following properties:

1. $\Lambda(z)$ is continuous in z.
2. $\lim_{z \to \left(\frac{\delta}{b\tau}\right)^{\frac{1}{a}}} \Lambda(z) = \rho\left(\frac{[(A-1)\delta - \delta_N]\left[1-\tau-\delta^{\frac{1}{a}}(b\tau)^{\frac{-(1-a)}{a}}\right]}{\Theta(\tau,b)}\right).$
3. $\lim_{z \to \frac{1-\tau}{b\tau}} \Lambda(z) = \breve{\rho}.$
4. $\lim_{z \to \left(\frac{\delta+\delta_N}{b\tau A}\right)^{\frac{1}{a}}} \Lambda(z) = \rho(0) = \breve{\rho}.$
5. $\frac{\vartheta \Lambda(z)}{\vartheta z} = \underbrace{\frac{\rho'(\cdot)b\tau}{\Theta(\tau,b)}}_{<0}[\underbrace{-(b\tau A z^a - \delta - \delta_N)}_{>0} + \underbrace{(1-\tau-b\tau z)aAz^{a-1}}_{>0}] < 0.$
6. $\frac{\vartheta^2 \Lambda(z)}{\vartheta z^2} = \underbrace{\frac{b\tau}{\Theta(\tau,b)}}_{<0}\{\underbrace{\rho''(\cdot)\frac{b\tau}{\Theta(\tau,b)}[-(b\tau A z^a - \delta - \delta_N) + (1-\tau-b\tau z)aAz^{a-1}]^{\mathbf{2}}}_{<0}$

$\underbrace{-\rho'(\cdot)aA[(1+a)b\tau z^{a-1}+(1-a)(1-\tau)z^{a-2}]}_{<0}\} > 0.$

Therefore, from 5 and 6 of $\Gamma(z)$ and $\Lambda(z)$ it follows that they both are strictly decreasing and convex functions. This implies that if an intersection exists, it can be unique or multiple. Then, assuming equilibrium existence, we have from 2-4 of $\Gamma(z)$ and $\Lambda(z)$ that if $a(1-\tau)A\left(\frac{\delta}{b\tau}\right)^{\frac{a-1}{a}} - (1-\sigma+\sigma A)\delta > \lim_{z\to\left(\frac{\delta}{b\tau}\right)^{\frac{1}{a}}} \Lambda(z)$, then a sufficient condition for more than one intersections is $A(a-\sigma)(1-\tau)^a(b\tau)^{1-a} - (1-\sigma)\delta > \check{\rho}$ under (i), or $-\sigma\delta_N - \delta + a(1-\tau)A\left(\frac{\delta+\delta_N}{b\tau A}\right)^{-\frac{(1-a)}{a}} > \check{\rho}$ under (ii). By contrast, if $a(1-\tau)A\left(\frac{\delta}{b\tau}\right)^{\frac{a-1}{a}} - (1-\sigma+\sigma A)\delta < \lim_{z\to\left(\frac{\delta}{b\tau}\right)^{\frac{1}{a}}} \Lambda(z)$, then a sufficient condition for more than one intersections is $A(a-\sigma)(1-\tau)^a(b\tau)^{1-a} - (1-\sigma)\delta < \check{\rho}$ under (i), or $-\sigma\delta_N - \delta + a(1-\tau)A\left(\frac{\delta+\delta_N}{b\tau A}\right)^{-\frac{(1-a)}{a}} < \check{\rho}$ under (ii). That is, if $\Gamma(z)$ starts above (below) $\Lambda(z)$, more than one intersections can exist when $\Gamma(z)$ also ends above (below) $\Lambda(z)$.

2 Transitional dynamics and stability analysis

Linearizing (11a)-(11c) around (12a)-(12c) implies that the local dynamics are approximated by the linear system:

$$\begin{bmatrix} \dot{\omega} \\ \dot{z} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} J_{\omega\omega} & J_{z\omega} & J_{x\omega} \\ J_{\omega z} & J_{zz} & J_{xz} \\ J_{\omega x} & J_{zx} & J_{xx} \end{bmatrix} \begin{bmatrix} \omega - \hat{\omega} \\ z - \hat{z} \\ x - \hat{x} \end{bmatrix}$$

where the elements of the Jacobian matrix, J, evaluated at the long run are:

$J_{\omega\omega} \equiv \frac{\vartheta\dot{\omega}}{\vartheta\omega} = \hat{\omega}\left[1 - \frac{\rho'(\cdot)\hat{z}\hat{x}}{1-\nu(1-\sigma)}\right] \gtreqless 0$

$J_{z\omega} \equiv \frac{\vartheta\dot{\omega}}{\vartheta z} = \frac{\hat{\omega}\left\{a(1-\nu)(1-\sigma)\Theta(\tau,b)A\hat{z}^a\hat{x} + (1-a)[1-\nu(1-\sigma)-a](1-\tau)A\hat{z}^{a-1} - \rho'(\cdot)\hat{\omega}\hat{z}\hat{x}\right\}}{[1-\nu(1-\sigma)]\hat{z}} \gtreqless 0$

$J_{x\omega} \equiv \frac{\vartheta\dot{\omega}}{\vartheta x} = \frac{\hat{\omega}}{1-\nu(1-\sigma)}[(1-\nu)(1-\sigma)\Theta(\tau,b)A\hat{z}^a - \rho'(\cdot)\hat{\omega}\hat{z}] < 0$

$J_{\omega z} \equiv \frac{\vartheta\dot{z}}{\vartheta\omega} = -\hat{z} < 0$

$J_{zz} \equiv \frac{\vartheta\dot{z}}{\vartheta z} = -(1-a)(1-\tau)A\hat{z}^{a-1} - ab\tau A\hat{z}^a < 0$

$J_{xz} \equiv \frac{\vartheta\dot{z}}{\vartheta x} = 0$

$J_{\omega x} \equiv \frac{\vartheta\dot{x}}{\vartheta\omega} = 0$

$J_{zx} \equiv \frac{\vartheta\dot{x}}{\vartheta z} = a\frac{\hat{x}}{\hat{z}}(\delta+\delta_N) > 0$

$J_{xx} \equiv \frac{\vartheta\dot{x}}{\vartheta x} = -\Theta(\tau,b)A\hat{z}^a\hat{x} > 0$

The trace and the determinant of J, $trace(J) = J_{\omega\omega} + J_{zz} + J_{xx}$ and $det(J) = J_{\omega\omega}J_{zz}J_{xx} - J_{z\omega}J_{\omega z}J_{xx} + J_{x\omega}J_{\omega z}J_{zx}$ have ambiguous signs. Due to the complexity for the computation of these signs, we provide numerical results for the eigenvalues of J, denoted by ε with regard to the long-run equilibria displayed in Table 2 of the paper. The findings, reported in Table A1, show that for the 'bad' equilibrium the dynamic system has two positive and one negative eigenvalues. Hence it follows that there exist locally a one-dimensional stable and a two-dimensional unstable manifolds, since we have one jump variable (ω) and two state/predetermined variables (z, x). However, in the 'good' equilibrium there are two negative and one positive eigenvalues, which implies that this regime is saddle-path stable.

3 The Social Planner problem

The Social Planner (SP) maximizes aggregate discounted utility subject to the production technology, the aggregate resource constraint and the laws of motion for the private and public capital stocks and environmental quality. Due to the variable RTP, Pontryagin's maximum principle cannot be applied directly. To solve the problem within the standard optimal control framework, we introduce an additional 'artificial' variable that accounts for the development of the accumulated discount rate, $\Delta(t) \equiv \int_0^t \rho(N_v, C_v)dv$. Formally, the SP problem is given by:

$$\max U_0 = \int_0^\infty u(C_t, N_t) \exp\left[-\Delta(t)\right] dt$$

subject to

$$\dot{K} = AK^a K_g^{1-a} - C - G - E - \delta K \tag{FB1}$$

$$\dot{K_g} = G - \delta K_g \tag{FB2}$$

$$\dot{N} = \theta E - sAK^a K_g^{1-a} + \delta_N N \tag{FB3}$$

$$\dot{\Delta} = \rho(N, C) \tag{FB4}$$

In contrast to the DCE, the externalities associated with the endogenous RTP as well as with the presence of the public capital stock in the production function and the environmental stock in the utility function are now internalized, since the SP solves for the aggregate quantities. The first-order conditions with respect to C, G, E, K, K_g, N, Δ are given by:

$$\nu C^{\nu(1-\sigma)-1} N^{(1-\nu)(1-\sigma)} e^{-\Delta} - \tilde{\lambda}_K + \tilde{\lambda}_\Delta \rho'(\cdot) N^{-1} = 0 \tag{FB5}$$

$$\tilde{\lambda}_K = \tilde{\lambda}_{K_g} \tag{FB6}$$

$$\tilde{\lambda}_K = \theta \tilde{\lambda}_N \tag{FB7}$$

$$-\dot{\tilde{\lambda}}_K = \left(\tilde{\lambda}_K - s\tilde{\lambda}_N\right) \alpha A K^{\alpha-1} K_g^{1-\alpha} - \tilde{\lambda}_K \delta \tag{FB8}$$

$$-\dot{\tilde{\lambda}}_{K_g} = \left(\tilde{\lambda}_K - s\tilde{\lambda}_N\right) (1-\alpha) A K^{\alpha} K_g^{-\alpha} - \tilde{\lambda}_{K_g} \delta \tag{FB9}$$

$$-\dot{\tilde{\lambda}}_N = (1-\nu) C^{\nu(1-\sigma)} N^{(1-\nu)(1-\sigma)-1} e^{-\Delta} + \tilde{\lambda}_N \delta_N - \tilde{\lambda}_\Delta \rho'(\cdot) C N^{-2} \tag{FB10}$$

$$\dot{\tilde{\lambda}}_\Delta = \frac{C^{\nu(1-\sigma)} N^{(1-\nu)(1-\sigma)} e^{-\Delta}}{1-\sigma} \tag{FB11}$$

where the dynamic multipliers $\tilde{\lambda}_K$, $\tilde{\lambda}_{K_g}$, $\tilde{\lambda}_N$, $\tilde{\lambda}_\Delta$ correspond to (FB1)-(FB4). The first-order conditions given by (FB5)-(FB11) and the transversality condition:

$$\lim_{t\to\infty} H_t^{SP} = 0 \tag{FB12}$$

complete the set of necessary conditions for welfare maximization.

To proceed we define $\lambda_i \equiv \tilde{\lambda}_i e^{\Delta}$ for $i = K, K_g, N, \Delta$. Then, using (FB6), (FB8) and (FB9), we obtain the familiar result that the ratio of the private to public capital stock evaluated at the optimum depends upon the ratio of the corresponding elasticities in the production function:

$$\left(\frac{K}{K_g}\right)^{SP} = \frac{\alpha}{(1-\alpha)} \tag{FB13}$$

At the BGP $\frac{\dot{C}}{C}=\frac{\dot{K}}{K}=\frac{\dot{K}_g}{K_g}=\frac{\dot{N}}{N}=g^{SP}$, which implies a constant value for the long-run RTP. Then from (FB11) we have that for $\frac{\dot{\lambda}_\Delta}{\lambda_\Delta}$ to be constant in the long run $u(C_t, N_t)$ and λ_Δ should grow at the same rates, which implies that $\frac{\dot{\lambda}_\Delta}{\lambda_\Delta}=(1-\sigma)g^{SP}$. Differentiating (FB5) with respect to time and using (FB7), (FB8) and (FB13), we obtain the balanced growth rate in the centrally planned economy:

$$g^{SP}(\bar{\omega}\bar{x})=\frac{1}{\sigma}\left[\alpha\left(1-\frac{s}{\theta}\right)A\left(\frac{\alpha}{(1-\alpha)}\right)^{\alpha-1}-\delta-\rho\left(\frac{\alpha}{(1-\alpha)}\bar{\omega}\bar{x}\right)\right] \tag{FB14}$$

The balanced growth rate is expressed here as a function of the long-run ratio of consumption to private capital, $\bar{\omega}$, and the ratio of public capital to environmental stock, $\bar{x}$.

Since we are dealing with an autonomous problem, the Hamiltonian is constant over time (e.g. Palivos et al., 1997). In conjunction with the transversality condition, this implies $H=0$ for all t. Using this in (FB9) and combining also (FB5), (FB7) and (FB13), we can derive from (FB10) $\bar{\omega}$ as a decreasing function of $\bar{x}$:

$$\bar{\omega}(\bar{x})=\frac{\nu(1-\alpha)^2(1-\sigma)\left[\alpha\left(1-\frac{s}{\theta}\right)A\left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1}-\delta-\delta_N\right]}{\alpha\bar{x}\left[\rho'(\cdot)(\theta\bar{x}+1-\alpha)+\theta(1-\nu)(1-\sigma)(1-\alpha)\right]} \tag{FB15}$$

which gives a well-defined (i.e. positive) solution for $\bar{\omega}$ if $\alpha\left(1-\frac{s}{\theta}\right)A\left(\frac{\alpha}{(1-\alpha)}\right)^{\alpha-1}-\delta-\delta_N>0$.

Finally, it follows from (FB1)-(FB3) and (FB13) that $\bar{x}$ at the BGP is determined by:

$$g^{SP}(\bar{\omega}\bar{x})\cdot\left(\frac{\bar{x}}{1-\alpha}+\frac{1}{\theta}\right)-\frac{1}{(1-\alpha)}\left[\alpha\left(1-\frac{s}{\theta}\right)A\left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1}-\delta-\alpha\bar{\omega}(\bar{x})\right]\bar{x}-\frac{\delta_N}{\theta}=0 \tag{FB16}$$

Once $\bar{x}$ is determined, (FB15) gives $\bar{\omega}(\bar{x})$ and in turn (FB14) provides $g^{SP}(\bar{\omega}(\bar{x})\bar{x})$. Notice that in contrast to the DCE, the balanced growth rate in the centrally planned economy depends on agents' environmental concerns in the utility function, $(1-\nu)$, since the solutions for $\bar{\omega}(\bar{x})$, $\bar{x}$ depend on ν.

For the exogenous RTP case, notice that the market economy can attain the optimal private-to-public-capital ratio ($z^{DCE}=z^{SP}$) and balanced growth rate ($g^{DCE}=g^{SP}$) if government policy is set as follows:

$$\tau = \frac{s}{\theta} \tag{FB17}$$

$$b = \frac{\alpha\left(1-\frac{s}{\theta}\right) A \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} - (1-\sigma)\delta - \rho}{\sigma \frac{s}{\theta} A \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}} \tag{FB18}$$

where (FB18) is derived by substituting (FB17) and (FB13) in (12c) of the paper. However, this would not be sufficient to also achieve $x^{DCE} = x^{SP}$ and $\omega^{SP} = \omega^{DCE}$, which suggests that the available policy instruments here are not sufficient to correct all the market failures and reproduce the first-best outcome. This is immediate visible by comparing (FB15)-(FB16) and (12a)-(12b) in the paper, since x^{SP} and ω^{SP} depend on an extra parameter, ν, which is not taken into account by (FB17)-(FB18). In the case of endogenous impatience, the growth rate in (FB14) depends additionally on x^{SP} and ω^{SP}, and hence through them on ν, which means that the first-best outcomes including the optimal growth rate cannot be replicated with the available tax-spending policy instruments.

4 Equations (16a)-(16b) in the Ramsey allocation

The Hamiltonian of the problem is given by:

$$H^R = \frac{\left(C^\nu N^{1-\nu}\right)^{1-\sigma}}{1-\sigma} e^{-\Delta} + \tilde{\lambda}_1 \left[(1-\tau)AK^a K_g^{1-a} - C - \delta K\right] + \tilde{\lambda}_2 \left(G - \delta K_g\right)$$
$$+ \tilde{\lambda}_3 \left(\delta_N N - sAK^a K_g^{1-a} + \theta E\right) + \tilde{\lambda}_4 \left(\tau AK^a K_g^{1-a} - G - E\right) + \tilde{\lambda}_5 \rho \left(\frac{C}{N}\right)$$

The optimality conditions, as given by equations (15a)-(15h), and the competitive-equilibrium growth rates, given by (9a)-(9d), completely characterize the solution of the Ramsey problem.

4.1 Derivation of (16a)

Using $\lambda_1 = \lambda_2 = \theta\lambda_3 = \lambda_4 \Rightarrow \dot{\lambda}_1 = \dot{\lambda}_2 = \theta\dot{\lambda}_3 = \dot{\lambda}_4$ in (15b) we get:

$$\dot{\lambda}_4 = -\lambda_4(1-a)(1-\tau)AK^a K_g^{-a} + \lambda_4\delta + \frac{1}{\theta}\lambda_4(1-a)sAK^a K_g^{-a} - \lambda_4(1-a)\tau AK^a K_g^{-a} + \lambda_4\rho(\cdot)$$

or equivalently:

$$\dot{\lambda}_4 = -\left(1 - \frac{s}{\theta}\right)\lambda_4(1-a)AK^aK_g^{-a} + \lambda_4\delta + \lambda_4\rho(\cdot) \tag{R1}$$

Then substituting (R1) and (9c) in $\dot{\chi} = \dot{\lambda}_4 K_g + \lambda_4 \dot{K}_g$ we obtain (16a) in the paper:

$$\frac{\dot{\chi}}{\chi} = -\left(1 - \frac{s}{\theta}\right)(1-a)Az^a + \rho(\cdot) + b\tau Az^a$$

4.2 Derivation of (16b)

From (15a) we have:

$$\nu C^{\nu(1-\sigma)-1}N^{(1-\nu)(1-\sigma)} - \lambda_1 + \frac{1}{N}\lambda_5\rho'(\cdot) = 0$$

Substituting (15d) and multiplying by C it follows:

$$\frac{C}{N}\lambda_5\rho'(\cdot) = \lambda_4 K_g \frac{K}{K_g}\frac{C}{K} - \nu C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)} = \chi\omega z - \nu C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)} \tag{R2}$$

From (15e)-(15f) we have $\lambda_3 = \frac{1}{\theta}\lambda_2$. Using this, (15b) implies:

$$\dot{\lambda}_2 = -\lambda_1(1-a)(1-\tau)AK^aK_g^{-a} + \lambda_2\delta + \frac{1}{\theta}\lambda_2(1-a)sAK^aK_g^{-a} - \lambda_4(1-a)\tau AK^aK_g^{-a} + \lambda_2\rho(\cdot)$$

and (15c) implies:

$$\dot{\lambda}_3 = -(1-\nu)C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)-1} - \lambda_3\delta_N + \lambda_5\frac{C}{N^2}\rho'(\cdot) + \lambda_3\rho(\cdot)$$

Then, combing the above we can write $\theta\dot{\lambda}_3 = \dot{\lambda}_2$ as:

$$\begin{aligned} &-\theta(1-\nu)C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)} - \theta\lambda_3\delta_N N + \theta\lambda_5\frac{C}{N}\rho'(\cdot) + \theta\lambda_3 N\rho(\cdot) \\ &= -\lambda_1 N(1-a)(1-\tau)AK^aK_g^{-a} + \frac{1}{\theta}\lambda_2 N(1-a)sAK^aK_g^{-a} - \lambda_4 N(1-a)\tau AK^aK_g^{-a} + \lambda_2 N\delta + \lambda_2 N\rho(\cdot) \end{aligned}$$

which using $\lambda_1 = \lambda_2 = \theta\lambda_3 = \lambda_4$ becomes:

$$
\begin{aligned}
-\theta(1-\nu)C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)} - \lambda_4 N\delta_N + \theta\lambda_5\frac{C}{N}\rho'(\cdot) = & -\lambda_4 N(1-a)AK^a K_g^{-a} \\
& + \frac{1}{\theta}\lambda_4 N(1-a)sAK^a K_g^{-a} + \lambda_4 N\delta
\end{aligned}
$$

Using the definitions for the transformed variables $x = \frac{K_g}{N}$ and $\chi \equiv \lambda_4 K_g$ we obtain:

$$
-\theta(1-\nu)C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)} + \frac{\chi}{x}\delta_N + \theta\lambda_5\frac{C}{N}\rho'(\cdot) = -(1-s)(1-a)Az^a\frac{\chi}{x} + \frac{\chi}{x}\delta
$$

Using (R2) this becomes:

$$
-\theta(1-\nu)C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)} + \frac{\chi}{x}\delta_N + \theta\chi\omega z - \theta\nu C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)} = -(1-s)(1-a)Az^a\frac{\chi}{x} + \frac{\chi}{x}\delta
$$

or equivalently:

$$
C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)} = \frac{1}{\theta}\left[\delta_N - \delta + (1-s)(1-a)Az^a\right]\frac{\chi}{x} + \chi\omega z \tag{R3}
$$

Also, from (15g) we have in the long run:

$$
\frac{\dot{\lambda}_5}{\lambda_5} = \frac{C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)}}{(1-\sigma)\lambda_5} + \rho(\cdot) = 0 \Rightarrow \lambda_5 = -\frac{C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)}}{(1-\sigma)\rho(\cdot)} \tag{R4}
$$

Combining (R2) and (R4) we get:

$$
-\frac{C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)}}{(1-\sigma)\rho(\cdot)}\frac{C}{N}\rho'(\cdot) = \chi\omega z - \nu C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)}
$$

which after some algebra becomes:

$$
C^{\nu(1-\sigma)}N^{(1-\nu)(1-\sigma)}\left[\nu(1-\sigma)\rho(\cdot) - x\omega z\rho'(\cdot)\right] - (1-\sigma)\rho(\cdot)\chi\omega z = 0
$$

Finally, substituting (R3) we derive (16b) in the paper:

$$\left[\frac{1}{\theta x}\left(\delta_N - \delta\right) + \frac{1}{\theta x}(1-s)(1-a)Az^a + \omega z\right]\left[\nu(1-\sigma)\rho(\cdot) - x\omega z\rho'(\cdot)\right] - (1-\sigma)\rho(\cdot)\omega z = 0$$

Table A1. Eigenvalues of the Jacobian matrix

	'Bad' equilibrium			'Good' equilibrium		
b	ε_1	ε_2	ε_3	ε_1	ε_2	ε_3
0.5	-	-	-	0.035	-0.001	-0.140
0.55	-	-	-	0.036	-0.003	-0.146
0.6	0.052	0.648	-0.757	0.046	-0.008	-0.153
0.65	0.045	0.579	-0.671	0.064	-0.013	-0.163
0.7	0.038	0.511	-0.591	0.090	-0.016	-0.178
0.75	0.029	0.439	-0.509	0.127	-0.017	-0.203
0.8	0.014	0.348	-0.410	0.185	-0.011	-0.251

Note: $\nu = 0.5$. See also Table 2 of the paper.

Table A2. 'Green spending reforms' in the DCE

b	$\hat{z}_1$	$\hat{z}_2$	$\hat{\omega}_1$	$\hat{\omega}_2$	$\hat{x}_1$	$\hat{x}_2$	$\hat{\rho}_1$	$\hat{\rho}_2$	g_1	g_2
0.5	0.093	-	1.386	-	1.787	-	0.657	-	0.030	-
0.52	0.108	0.813	1.278	0.319	1.521	0.031	0.597	0.032	0.037	0.144
0.54	0.127	0.665	1.164	0.381	1.264	0.086	0.534	0.071	0.044	0.133
0.56	0.155	0.531	1.038	0.457	1.005	0.160	0.463	0.119	0.054	0.122
0.58	0.208	0.388	0.869	0.577	0.702	0.291	0.366	0.192	0.070	0.105
0.6	0.288	0.299	0.612	0.674	0.291	0.420	0.219	0.257	0.093	0.084

Note: $\tau = 0.45,\ A = 0.8,\ \gamma = 2.8\ \sigma = 1.3.$

See Table 1 of the paper for the rest of the parameter values.